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ERGODIC PROPERTIES OF EIGENFUNCTIONS FOR THE DIRICHLET PROBLEM

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Introduction, statement of the result, and notation. Let Ω be a bounded convex open subset of $\mathbb{R}^n$, such that the boundary has $W^{2,\infty}$ regularity, i.e., the unit normal vector field has Lipschitz regularity. Recall that the billiard in Ω is the dynamical system on the unit (co)tangent bundle on $\overline{\Omega}$ generated by the motion of a point in Ω along a geodesic with unit speed, with elastic reflections on the boundary—which amounts to identifying, above $\partial\Omega$, the symmetric vectors with respect to the tangent space to $\partial\Omega$. To be precise, the latter definition determines the trajectory of only *almost* every tangent vector, for the Liouville measure λ—namely, vectors not tangent to $\partial\Omega$ and such that the series of successive time intervals between two reflections does not converge (see for instance [KS], [Ha]). Anyway, this yields a (almost-everywhere-defined) one-parameter group (G_t) of measurable transformations leaving invariant the Liouville measure λ. The ergodicity of such a dynamical system was studied by several authors, particularly Bunimovitch [B], who gave first examples of ergodic convex billiards. The most famous example is the "stadium" in $\mathbb{R}^2$, or in $\mathbb{R}^3$ the region which appears under rotation of the stadium around its diameter.

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