

## ERGODIC PROPERTIES OF EIGENFUNCTIONS FOR THE DIRICHLET PROBLEM

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**Introduction, statement of the result, and notation.** Let  $\Omega$  be a bounded convex open subset of  $\mathbb{R}^n$ , such that the boundary has  $W^{2,\infty}$  regularity, i.e., the unit normal vector field has Lipschitz regularity. Recall that the billiard in  $\Omega$  is the dynamical system on the unit (co)tangent bundle on  $\bar{\Omega}$  generated by the motion of a point in  $\Omega$  along a geodesic with unit speed, with elastic reflections on the boundary—which amounts to identifying, above  $\partial\Omega$ , the symmetric vectors with respect to the tangent space to  $\partial\Omega$ . To be precise, the latter definition determines the trajectory of only *almost* every tangent vector, for the Liouville measure  $\lambda$ —namely, vectors not tangent to  $\partial\Omega$  and such that the series of successive time intervals between two reflections does not converge (see for instance [KS], [Ha]). Anyway, this yields a (almost-everywhere-defined) one-parameter group  $(G_t)$  of measurable transformations leaving invariant the Liouville measure  $\lambda$ . The ergodicity of such a dynamical system was studied by several authors, particularly Bunimovitch [B], who gave first examples of ergodic convex billiards. The most famous example is the “stadium” in  $\mathbb{R}^2$ , or in  $\mathbb{R}^3$  the region which appears under rotation of the stadium around its diameter.

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