Vol. 71, No. 2 DUKE MATHEMATICAL JOURNAL © August 1993

EFFECTIVE BOUND FOR THE GEOMETRIC LANG CONJECTURE

ALEXANDRU BUIUM

Introduction. Let A be an abelian variety over the complex field $\mathbf{C}$ and let $X \subset A$ be a closed subvariety. For any subset $M \subset X$ let us define $\lambda(M) \in \mathbf{N} \cup \{\infty\}$ as the minimum of the natural numbers n such that there exist abelian subvarieties $B_i \subset A$ and points $a_i \in A$ with

$$M \subset \bigcup_{i=1}^{n} (B_i + a_i) \subset X$$

if such an n exists; if no such n exists, we set by definition $\lambda(M) = \infty$.

Note that $\lambda(M)$ is at most the cardinality $\#(M)$ (with equality holding if X contains no translate of a nonzero abelian subvariety).

A celebrated conjecture of S. Lang [La] asserted that $\lambda(X \cap \Gamma) < \infty$ for any finite-rank subgroup $\Gamma \subset A$ (by definition the rank of Γ is the number $dim_{\mathbf{Q}}(\Gamma \otimes_{\mathbf{Z}} \mathbf{Q})$; we emphasize that Γ need not be finitely generated; in particular, it may contain A_{tors}). Lang's conjecture was finally proved through the work of Flatings [F] combined with work of Hindry [Hi]; the work on this conjecture has a long history which will not be recalled here.

It is natural to ask whether it is possible to bound explicitly the number $\lambda(X \cap \Gamma)$ in terms of the numbers g, r, s, d, e where g is the dimension of A, r is the rank of Γ, s is the dimension of X, d^2 is the degree of a fixed polarisation on A, and e is the degree of X with respect to this polarisation. In particular, we ask for a bound depending only on g for the number of torsion points on a curve of genus $g \geqslant 2$; by "torsion points" we mean, as usual, points of the curve which are torsion points of the Jacobian.

In this paper we find such a bound in the case when X is smooth and either A has $\mathbf{C}/\overline{\mathbf{Q}}$-trace zero or X is a curve not descending to $\overline{\mathbf{Q}}$. Recall that "$\mathbf{C}/\overline{\mathbf{Q}}$-trace zero" means "containing no nonzero abelian subvariety which descends to $\overline{\mathbf{Q}}$ = field of algebraic numbers". So roughly speaking, we solve the problem in the "geometric" (= "nonarithmetic") case.

One should note that in the case of Mordell's conjecture (X a curve, A its Jacobian, $\Gamma = A(K)$ the Mordell-Weil group, K a number field or a function field), one disposes of a series of effective results; cf. [Pa], [Sz1], [Sz2], [EV]. Yet these results have a quite different flavour: they bound heights in terms of invariants of K and in terms of the locus of "bad reduction". Finally note that one disposes of

Received 4 November 1992.