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DENSITY THEOREMS FOR CONGRUENCE GROUPS IN REAL RANK 1

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1. Introduction. Let G be a semisimple Lie group of noncompact type defined over $\mathbf{Q}$, and $\Gamma \subset G$ an arithmetic lattice in G. By "arithmetic" we understand that there is a rational embedding $\tau: G \hookrightarrow GL_n(\mathbf{R})$ such that $\tau(\Gamma)$ is commensurable with $G(\mathbf{Z}) = \tau(G) \cap GL_n(\mathbf{Z})$. From this embedding we further obtain *congruence* subgroups $\Gamma(q) \subset \Gamma$ by setting

$$\Delta(q) = \{\gamma \in G(\mathbf{Z}): \gamma \equiv I \pmod{q}\}$$

and

$$\Gamma(q) = \tau^{-1}(\tau(\Gamma) \cap \Delta(q)).$$

In what follows we will identify G with $\tau(G)$, Γ with $\tau(\Gamma)$, and $\Gamma(q)$ with $\tau(\Gamma) \cap \Delta(q)$ unless it leads to ambiguity. With this in mind for $g \in G$ set

$$\|g\|^2 = \mathrm{Tr}({}^t g g),$$

and for dg Haar measure on G let

$$\alpha = \lim_{T \to \infty} \left(\frac{\log \int_{\|g\| \leqslant T} dg}{\log T} \right).$$

For any lattice $\Delta \subset G$ define the lattice point counting function $N(T, \Delta)$ by

$$N(T, \Delta) = \#\{\delta \in \Delta: \|\delta\| \leqslant T\}.$$

Duke, Rudnick, and Sarnak [5] show that $N(T, \Gamma) \sim cT^\alpha$ as $T \to \infty$, and more pertinently for our discussion, that

$$N(T, \Gamma(q)) \sim \frac{cT^\alpha}{\iota(q)} \qquad \text{as } T \to \infty, \tag{1}$$

where $\iota(q)$ is the index $[\Gamma : \Gamma(q)]$.

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