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QUALITATIVE PROPERTIES OF SOLUTIONS TO SOME NONLINEAR ELLIPTIC EQUATIONS IN R^2

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0. Introduction. In this paper, we investigate properties of the solutions to the elliptic equations

$$-\Delta u = R(x)e^{u(x)} \qquad x \in R^2 \tag{$*$}$$

for functions $R(x)$ which are positive near infinity.

Equations of this kind arise from a variety of situations, such as from prescribing Gaussian curvature in geometry [2] and from combustion theory in physics [3].

Recently, a series of works have been sought to understand the existence and the qualitative properties of the solutions of ($*$). Ni [5] and Ni & Cheng [4] considered the case where $R(x)$ is nonpositive; McOwen [6] and Aviles [7] investigated the situation where $R(x) \to 0$ in some order as $|x| \to \infty$. In our previous paper [1], we consider a special case where R is a constant. We proved that the solutions are radially symmetric and, hence, classified all the solutions.

In this paper, we consider more general functions $R(x)$. First, we obtain the asymptotic behavior of the solution near infinity. Consequently, we prove that all the solutions satisfy an identity, which is somewhat of a generalization of the well-known Kazdan-Warner condition. Finally, using the asymptotic behavior together with the further development of the method employed in our previous paper [1], we show that all the solutions are radially symmetric provided R is radially symmetric and nonincreasing. This part can be viewed as the completion of [1].

Throughout this paper, we assume that the function $R(x)$ is positive near infinity.

In §1, we study the asymptotic behavior of the solution $u(x)$ of ($*$). Let $\beta = 1/2\pi \int_{R^2} R(x)e^{u(x)}\,dx$. Under some appropriate conditions, we show that the solutions approach $-\infty$ at the rate $-\beta \ln |x|$, and the value of β depends on the monotonicity of the function $R(x)$ in the radial direction. More precisely, we prove the following theorems.

THEOREM 1. *Assume that $R(x)$ is a bounded function and u is a solution of ($*$) with*

$$\int_{R^2} e^{u(x)}\,dx < \infty.$$

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