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WHEN IS A FAMILY OF SUBMANIFOLDS LOCALLY DIFFEOMORPHIC TO A FAMILY OF PLANES?

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1. Formulation and discussion of main results. On an n-dimensional manifold B let a family of k-dimensional submanifolds B_ξ parametrized by a *connected* manifold Γ be given.

1.1. Definition. We will say that a family of k-dimensional submanifolds of a domain U rectifies to the mth order if for any $x \in U$ there exists a diffeomorphism f of a neighborhood of x onto a domain of R^n identifying the manifold of m-jets of submanifolds of this family at x with an open domain in the manifold of m-jets of k-dimensional planes at $f(x)$.

The main result of this paper is the proof of the following theorem.

1.2. Theorem. *If a family Γ of k-dimensional submanifolds for $k > 1$ of an n-dimensional manifold B rectifies to the 2nd order, then it is locally diffeomorphic to a family of k-planes in R^n.*

The converse statement is obvious.

In the simplest case $k = 2$, $n = 3$, Theorem 1.2 turns into the following.

1.3. Proposition. *If a family of surfaces of R^3 rectifies to the 2nd order, then it is locally diffeomorphic to a family of planes in R^3.*

For complex manifolds a much stronger result holds:

1.4. Theorem ([GeGo]). *In the category of complex analytical manifolds, a family of k-dimensional submanifolds of an n-dimensional manifold B is locally isomorphic to the family of all k-dimensional planes in CP^n for $k > 1$ if and only if at any point $x \in B$ every k-dimensional subspace of T_xB is the tangent space for exactly one submanifold from the family.*

Theorem 1.2 is formulated in [GeGo] where the scheme of its proof is hinted for $k = 2$, $n = 3$. Here we will give its proof based on different ideas.

Theorem 1.4 is deduced from Theorem 1.2 (see [GeGo]). For completeness we deduce Theorem 1.4 from Theorem 1.2 in a trifle simpler way.

For $k = 1$ Theorems 1.2 and 1.4 are false. The counterexample is a family of geodesics for any projective connections. The following lemma shows that there are no other counterexamples.

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