Vol. 71, No. 1 DUKE MATHEMATICAL JOURNAL © July 1993

# ASYMPTOTICS OF THE DETERMINANT OF THE LAPLACIAN ON HYPERBOLIC SURFACES OF FINITE VOLUME

ROLF E. LUNDELIUS

## 1. Preliminaries.

*1.1. Introduction and summary of results.* In this paper we consider one aspect of the general question, "What geometric information about a Riemannian manifold can be determined from the spectrum of the Laplace operator on that manifold?" The problem has been studied in many settings, and results have been obtained which relate the spectrum of the Laplacian to dimension, topology, volume, diameter, curvature, and other geometric quantities. For an introduction to this subject, see [Ka]. The spectrum of the Laplacian on compact manifolds consists of discrete eigenvalues with finite multiplicity. In the compact case the problem of writing down a formula for the eigenvalues of the Laplacian seems intractable for all but a very few manifolds. Therefore, most of these studies have focused on a very small amount of spectral data, typically, some small number of the lowest eigenvalues or the asymptotic growth of eigenvalues. For noncompact manifolds, the situation is a little worse. In this case, the spectrum may consist of a continuous and a discrete part. Considered as a *set*, the continuous spectrum contains very little spectral information; the important information is contained in a measure on that set. The very existence of discrete spectrum embedded in the continuous spectrum is poorly understood.

The *determinant of the Laplacian* of a compact manifold is a spectral invariant which depends on every eigenvalue. For convenience, we discuss the negative logarithm of the determinant, which we call the *height*. The height was originally defined as a special value of the analytic continuation of a sum over eigenvalues. For orientable, two-dimensional, smooth Riemannian manifolds (henceforth called *surfaces*), there is a formula relating the height of one surface to the height of any conformally equivalent surface. This formula, due to Polyakov, has been used to show that surfaces of constant scalar curvature extremize the height [OPS]. Complementing this result is a theorem of Wolpert which gives the asymptotic behavior of the height on degenerating hyperbolic surfaces [Wo]. In this paper we generalize the definition of the height and these two results to a class of noncompact surfaces.

We define a class of *admissible surfaces* to consist of compact surfaces with a finite number (possibly zero) of finite volume hyperbolic cusps glued in. See Section 2.1

Received 21 May 1992. Revision received 8 January 1992.