Vol. 71, No. 1 DUKE MATHEMATICAL JOURNAL © July 1993

MIXING, COUNTING, AND EQUIDISTRIBUTION IN LIE GROUPS

ALEX ESKIN AND CURT MCMULLEN

1. Introduction. Let $\Gamma \subset G = \mathrm{Aut}(\mathbb{H}^2)$ be a group of isometries of the hyperbolic plane $\mathbb{H}^2$ such that $\Sigma = \Gamma\backslash\mathbb{H}^2$ is a surface of finite area. Then:

I. The geodesic flow is mixing on the unit tangent bundle $T_1(\Sigma) = \Gamma\backslash G$.

II. The sphere $S(x, R)$ of radius R about a point $x \in \Sigma$ becomes equidistributed as $R \to \infty$.

III. The number of points $N(R)$ in an orbit Γv which lie within a hyperbolic ball $B(p, R) \subset \mathbb{H}^2$ has the asymptotic behavior

$$N(R) \sim \frac{\mathrm{area}(B(p, R))}{\mathrm{area}(\Sigma)}.$$

(See §2 for more detailed statements).

The purpose of this paper is to discuss results similar to those above where the hyperbolic plane is replaced by a general *affine symmetric space* $V = G/H$. This setting includes the classical Riemannian symmetric spaces (when H is a maximal compact subgroup) as well as spaces with *indefinite* invariant metrics.

A simple non-Riemannian example is obtained by letting V be the space of oriented geodesics in the hyperbolic plane. Then $H = A$, the group of diagonal matrices in $G = PSL_2(\mathbb{R})$. In this case $\Gamma\backslash G/H$ is not even Hausdorff.

This setting includes counting theorems for integral points on a large class of homogeneous varieties (e.g. those associated to quadratic forms) and allows us to prove some of the main theorems of [DRS] by elementary arguments (see §6).

Statement of Results. Let G be a connected semisimple Lie group with finite center and let $H \subset G$ be a closed subgroup such that G/H is an *affine symmetric space* (cf. [F-J], [Sch]). This means there is an involution $\sigma\colon G \to G$ such that H is the fixed-point set of σ:

$$H = \{g\colon \sigma(g) = g\}.$$

(By *involution* we mean a Lie group automorphism such that $\sigma^2 = \mathrm{id}$.)

Let $\Gamma \subset G$ be a *lattice*, i.e. a discrete subgroup such that the volume of $X = \Gamma\backslash G$ is finite.

Received 11 May 1992. Revision received 15 January 1993.
McMullen's research partially supported by the NSF and the Alfred P. Sloan Foundation.