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SINGULARITIES OF THE SOLUTION TO A CERTAIN CAUCHY PROBLEM AND AN APPLICATION TO THE POMPEIU PROBLEM

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0. Introduction. This paper consists of two parts. The first is a report on an integral transform of B. Yu. Sternin and V. E. Shatalov and its application to the holomorphic Cauchy problem (see Section 1) with constant coefficients. The reason for including this is to collect some of their results on the Cauchy problem in one paper. At present, these are scattered in a number of papers (see the references). Also, the author has been unable to find any explicit theorems like Theorem 1.1 below.

The second part is an application of the first to the Pompeiu problem (see Section 2). The main result is Theorem 2.2, which implies that under certain hypotheses on a domain Ω_0 in $\mathbb{R}^2$, symmetric about the x_1-axis, the rotation of Ω_0 around the x_1-axis in $\mathbb{R}^3$, denoted by Ω, has the Pompeiu property; i.e. $f \equiv 0$ is the only continuous function in $\mathbb{R}^3$ such that

$$\int_{\sigma(\Omega)} f = 0 \tag{0.1}$$

for every rigid motion σ of $\mathbb{R}^3$. In fact, a stronger property of Ω is shown to hold: a certain family of overdetermined Cauchy problems (see (2.3)) has no solutions. At this point, we should point out (as the referee so kindly remarked) that, in order to deduce the Pompeiu property for Ω, we only need to consider a smaller family of Cauchy problems (see (2.2)), all of which are rotation invariant with respect to the x_1-axis. Consequently, the solutions are axially symmetric. It is well known (see [M]) that the conclusion, deduced from Theorem 1.1, in the paragraph following Lemma 2.2 holds for the singularities of such functions. In fact, an analogous conclusion holds in any number of dimensions. As a consequence, the proofs of Lemma 2.3 and Theorem 2.2 (for the problem (2.2)) can be carried through in any number of dimensions. This means that we may deduce the Pompeiu property for the rotation of Ω_0 in $\mathbb{R}^n$, for any n (see the concluding remarks in Section 3). Thus, the reader interested only in the Pompeiu problem may skip Section 1 altogether. However, in order to deduce the stronger property mentioned above, we need Theorem 1.1, and for the problem at hand, this theorem can only be applied in three dimensions.

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