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CENTRALLY TRIVIAL AUTOMORPHISMS AND AN ANALOGUE OF CONNES'S $\chi(M)$ FOR SUBFACTORS

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1. Introduction. The aim of this paper is to exploit the notion of centrally trivial automorphisms for subfactors. We give an effective upper bound for the size of these automorphisms and their complete characterization with an additional assumption for finite-depth subfactors, introduce and compute $\chi(M, N)$, the relative version of Connes's invariant $\chi(M)$, discuss their analogy to modular automorphism groups of (injective) type-III factors, and give some applications on $\mathrm{Aut}(M, N)$.

Since the breakthrough of V. Jones on index for subfactors [J3], the importance of the study of subfactors both in operator algebra theory itself and in other fields of mathematics has become clearer and clearer. Our aim here is to extend the Connes-type automorphism approach [C1, C2, C3, C4, C5] to a subfactor setting, which is a natural continuation of our orbifold construction in [EK, IK, Ka2, Ka3].

A. Ocneanu [O2] introduced a notion of *paragroup* as a combinatorial characterization of higher relative commutants of approximately finite-dimensional (AFD) subfactors of type II_1 with trivial relative commutant, finite index, and finite depth. (See also [Ka2, Ka3, O3, O4].) Because S. Popa [P2, P3, P4] has proved that the higher relative commutants generate the original subfactor in a very general condition called strong amenability, combinatorial approach of paragroup gives a satisfactory classification in many cases. In particular, Ocneanu's announcement of classification of subfactors with index less than 4 without a full proof has been verified by [I1, I3, Ka2, SV], and a classification for the case of index equal to 4 was also obtained by [IK, P3]. (In the case of E_6, we use an earlier construction of Bion-Nadal [BN].)

A paragroup has a certain algebraic structure on two graphs. Conceptually, it can be regarded as a quantization of a Galois group, and technically as a discrete analogue of a compact manifold. In particular, an analogue of a flat connection plays a key role. Furthermore, Ocneanu [O5] recently announced that certain complex-number–valued topological invariants of 3-dimensional manifolds are in bijective correspondence to paragroups. Thus it is has a very deep and rich mathematical structure, but for our purpose here, the most important aspect of paragroup theory is its relation to statistical mechanics. As mentioned in [O2] and explicitly clarified in [EK, Ka3], a paragroup is quite similar to exactly solvable lattice models (IRF models) without a spectral parameter. (See [ABF, Ba, DJMO, Ji] for IRF models.) The commuting square condition in operator algebra theory corresponds

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