Vol. 71, No. 1 DUKE MATHEMATICAL JOURNAL © July 1993

SPLITTING OF NONNEGATIVELY CURVED LEAVES IN MINIMAL SETS OF FOLIATIONS

SCOT ADAMS AND GARRETT STUCK

0. Introduction. The main result of [AF1] states:

THEOREM A [AF1, Theorem 5.1, p. 695]. *Let M be a finite measure space. Let $\mathscr{F}$ be a measure-preserving foliation of M by Riemannian manifolds of nonnegative Ricci curvature. Then almost every leaf of M is isometric to the product of a compact manifold by a flat Euclidean space.* ■

In the sequel, if L is a Riemannian manifold which is isometric to such a product, then we will simply say that "L splits".

It is natural to inquire whether the conclusion of Theorem A can be strengthened for smooth foliations of compact manifolds. Let $\mathscr{F}$ be a smooth foliation of a compact Riemannian manifold and suppose every leaf has nonnegative Ricci curvature (in the inherited metric). Theorem A implies that almost every leaf in the support of a transverse invariant measure splits. We are interested in extending Theorem A to obtain information about *every* leaf in the support of a transverse invariant measure. (One cannot expect an arbitrary leaf to split. For example, there are metrics on S^3 for which the leaves of the Reeb foliation have nonnegative Ricci curvature, but for such metrics the noncompact leaves do not split and are not contained in the support of a transverse invariant measure.)

Recall that a minimal set in a foliated manifold is a minimal element in the collection of closed, leaf-saturated subsets of the manifold, ordered by inclusion. It is a result of J. Plante [Pla1, Theorem 4.1, p. 339] that, if a leaf L has nonexponential growth, the closure of L supports a transverse invariant measure of finite total volume. If L is also contained in a minimal set $\mathscr{L}$, it follows that $\mathscr{L}$ is exactly the support of such a measure. Now if L has nonnegative Ricci curvature, then every leaf in $\mathscr{L}$ has nonexpontential growth, and so Theorem A implies that almost every leaf in $\mathscr{L}$ must split. Since every foliation of a compact manifold contains a minimal set, we have the following statement:

COROLLARY B. *If a foliation of a compact manifold contains a leaf of nonnegative Ricci curvature, then it contains a leaf which is isometric to the Riemannian product of a compact manifold* (*of nonnegative Ricci*) *and a flat Euclidean space.*

It does not follow from these observations that: if L is an *arbitrary* leaf contained in a minimal set and if L has nonnegative Ricci curvature, then L must split. (See

Received 23 May 1992. Revision received 30 November 1992.
Both authors were supported by NSF Postdoctoral Research Grants.