Vol. 71, No. 1 DUKE MATHEMATICAL JOURNAL © July 1993

ON THE ASYMPTOTIC BEHAVIOR OF SPHERICALLY SYMMETRIC WAVE MAPS

DEMETRIOS CHRISTODOULOU AND A. SHADI TAHVILDAR-ZADEH

1. Introduction. Wave maps are maps from an Einsteinian manifold M, i.e. a manifold with a metric g of index 1, into a Riemannian manifold N, i.e. one with a positive definite metric h. Formally, they are the critical points $U: M \to N$ of the Lagrangian

$$\mathscr{L}[U] = \frac{1}{2} \int_M h_{ab} \partial_\mu U^a \partial^\mu U^b.$$

In a previous paper [1] we proved the global regularity of the solution to the Cauchy problem for a spherically symmetric wave map in the case where M is the $(2 + 1)$-dimensional Minkowski space-time and N satisfies the following two conditions.

(C1) *There exists an orthonormal frame of smooth vectorfields Ω_A on N whose structure functions e_{AB}^C are bounded.*

(C2) *For each $p \in N$, let $\Sigma(p, s)$ be the geodesic sphere of radius s centered at p and let k_{AB} be its second fundamental form. Then there exist constants c and C such that*

$$s\lambda \geqslant c \quad \textit{and} \quad s\Lambda \leqslant C(1 + s),$$

where λ and Λ are respectively the smallest and largest eigenvalues of k_{AB}.

The map U satisfies the Euler-Lagrange system that in local coordinates can be written as

$$\Box U^a + \Gamma^a_{bc} \partial_\mu U^b \partial^\mu U^c = 0, \tag{1}$$

where $\Box = -\partial_t^2 + \partial_r^2 + r^{-1}\partial_r$, and Γ's are the Christoffel symbols of the target N.

Here we obtain the asymptotic behavior of these solutions. In particular, it will be shown that the asymptotic behavior is like that of the linear wave equation. We will use the notation of [1] to the extent possible. The main result is the following one.

THEOREM 1. *Let $\mathscr{C}_u^+$ (resp. $\mathscr{C}_u^-$) be the interior of the future (resp. past) light cone with vertex at $(t = u, r = 0)$ in $M = \mathbb{R}^{2,1}$. Then for U a smooth, spherically symmetric*

Received 23 July 1992. Revision received 15 December 1992.
Christodoulou supported in part by NSF grant PHY 8815069.
Tahvildar-Zadeh supported by NSF grant DMS 9100383.