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A COMPLETE MINIMAL KLEIN BOTTLE IN $\mathbb{R}^3$

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1. Introduction. Classical examples of complete minimal surfaces in three-dimensional Euclidean space have been important in the development of this field of geometry.

The catenoid, Enneper's surface (see [Os]), or Chen-Gackstatter's surface [CG] are examples of low total curvature which admit nice characterizations (see [Os], [LR], [L], and [Bl]).

However, in the nonorientable case, we do not have a lot of examples. For a long time, the only one known was Henneberg's surface, which is not regular (see for instance [SP]). Later, Meeks exhibited a regular example with total curvature -6π and gave a uniqueness theorem for this surface (see [M]).

Nonorientable minimal surfaces are rather elusive, and today we know only examples with high total curvature and relatively low topology (from the point of view of Jorge-Meeks's formula). See for instance [Ba], [I1], [I2], [Ol], [T], [K1], [K2], and [S].

In this paper we show a complete minimal Klein bottle in $\mathbb{R}^3$ with an Enneper end and total curvature -8π, which is the least admissible upper bound.

Geometrically, this surface is obtained by attaching a Klein bottle to the Enneper surface.

In a forthcoming paper, the author will publish a uniqueness theorem, characterizing his example as the only complete nonorientable minimal surface in $\mathbb{R}^3$ with total curvature -8π.

Complete nonorientable minimal surfaces in $\mathbb{R}^3$. Let $x\colon M \to \mathbb{R}^3$ be a complete nonorientable minimal surface in $\mathbb{R}^3$ with finite total curvature.

Let $\pi_0\colon M_0 \to M$ be the conformal oriented two-sheeted cover of M, and take $I\colon M_0 \to M_0$ to be the antiholomorphic order-two deck transformation for this cover. Then $x_0 = x \circ \pi_0$ is a minimal immersion of an orientable surface, and (g, η) represents its Weierstrass data (see [Os]).

It is well known that, modulo natural identifications, g is the Gauss map of M_0 and

$$x_0 = \mathrm{Real} \int (\phi_1, \phi_2, \phi_3) \tag{1}$$

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