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THE CAUCHY PROBLEM FOR THE KORTEWEG-DE VRIES EQUATION IN SOBOLEV SPACES OF NEGATIVE INDICES

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1. Introduction. In this paper we study the initial value problem (IVP) for the Korteweg-de Vries equation

$$\begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial^3 u}{\partial x^3} + \dfrac{\partial(u^2)}{\partial x} = 0 & t, x \in \mathbb{R}, \\ u(x, 0) = u_0(x). \end{cases} \tag{1.1}$$

We are interested in well-posedness results for data in classical Sobolev spaces of negative order, i.e. $u_0 \in H^{-s}(\mathbb{R})$, with $s \geqslant 0$. Here the notion of well-posedness includes the existence, uniqueness, persistence property (i.e. the solution describes a continuous curve in X whenever $u_0 \in X$) and the continuous dependence of the solution upon the data.

Our main result (see Theorem 1 below) shows that (1.1) is locally well posed in $H^{-s}(\mathbb{R})$ for $s < 5/8$. Hence, taking $s \in (1/2, 5/8)$, it follows that the IVP (1.1) has a unique local solution for any bounded measure u_0. In particular, for $u_0 = \delta$ this answers (locally) the uniqueness questions left open by Y. Tsutsumi [11, Remark 3.1].

The IVP (1.1) has been considered in many works. On one hand, there are existence results of weak solutions corresponding to rough data. Using the inverse scattering method, T. Kappeler [4] showed that if u_0 is a real measure satisfying an appropriate decay at infinity, then (1.1) has a global solution. In [5] T. Kato established global existence results for data $u_0 \in L^2$ (see also [9]). In [11] Y. Tsutsumi proved the existence of a global solution of (1.1) for initial data a positive measure. His argument combines the smoothing effects and the existence results deduced in [5] for the modified KdV and the Miura transformation. On the other hand, there are well-posedness results. In this direction, J. Bourgain [2] has recently shown that (1.1) is locally well posed in L^2. Due to the second conservation law, this result extends globally in time. Previous results guaranteed that (1.1) is locally and globally well posed in H^s with $s > 3/4$ and $s \geqslant 1$ respectively (see [7]). For further references and comments see [8].

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