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ABELIAN QUOTIENTS OF SUBGROUPS OF THE MAPPING CLASS GROUP OF SURFACES

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1. Introduction. Let Σ_g be a closed oriented surface of genus g and let $\mathcal{M}_g$ be its mapping class group. Namely, it is the group consisting of all the isotopy classes of orientation-preserving diffeomorphisms of Σ_g. The mapping class group plays very important roles in several branches of mathematics, e.g. the low-dimensional topology, the theory of algebraic curves, the theory of Teichmüller spaces, the modern quantum field theory, and in particular, the conformal field theory on compact Riemann surfaces. However, the structure of the mapping class group is far from being well understood.

The mapping class group $\mathcal{M}_g$ acts on the homology group $H_1(\Sigma_g; \mathbf{Z})$ of Σ_g, and this action gives rise to the classical representation

$$\mathcal{M}_g \to \mathrm{Sp}(2g; \mathbf{Z})$$

of $\mathcal{M}_g$ onto the Siegel modular group $\mathrm{Sp}(2g; \mathbf{Z})$, which is the group consisting of all the symplectic transformations of $H_1(\Sigma_g; \mathbf{Z})$ with respect to the skew-symmetric pairing on it induced by the intersection number. The kernel of this representation, denoted by $\mathcal{I}_g$, is called the Torelli group. Extending an earlier work by Sullivan, Johnson [5], [6], [7] obtained fundamental results concerning the structure of the Torelli group. In particular, he determined the abelianization of $\mathcal{I}_g$ in [7]. In our paper [15], generalizing the above mentioned works by Sullivan and Johnson in the context of the whole mapping class group rather than just the Torelli group, we began a systematic study of the structure of $\mathcal{M}_g$. This can be summarized roughly as follows. For simplicity, here we consider only the mapping class group $\mathcal{M}_{g,1}$ of Σ_g *relative* to an embedded disc $D^2 \subset \Sigma_g$. Then $\mathcal{M}_{g,1}$ acts naturally on $\Gamma_1 = \pi_1(\Sigma_g \backslash D^2)$, which is a free group of rank $2g$, and the induced representation $\mathcal{M}_{g,1} \to \mathrm{Aut}\, \Gamma_1$ is classically known to be injective. Now let $\{\Gamma_k\}_{k \geqslant 1}$ be the lower central series of Γ_1, namely $\Gamma_{k+1} = [\Gamma_k, \Gamma_1]$ for $k \geqslant 1$. Then $\mathcal{M}_{g,1}$ acts on the successive nilpotent quotients $N_k = \Gamma_1/\Gamma_k$ by automorphisms so that we have a series of representations

$$\rho_k\colon \mathcal{M}_{g,1} \to \mathrm{Aut}\, N_k \qquad (k = 2, 3, \ldots).$$

N_2 is canonically isomorphic to $H_1(\Sigma_g; \mathbf{Z})$, and ρ_2 is the same as the classical representation $\mathcal{M}_g \to \mathrm{Sp}(2g; \mathbf{Z})$ mentioned above. We can consider the images of

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