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THE ALGEBRAIC K-THEORY OF THE CLASSICAL GROUPS AND SOME TWISTED FORMS

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Introduction. Merkurjev and Suslin [MS] pioneered the use of the higher algebraic K-theory of varieties as a tool for understanding the Milnor K-theory of fields and its relation with Galois cohomology. The varieties in question are the twisted forms of projective spaces, the so-called Brauer-Severi varieties. A Brauer-Severi variety V over a field k is associated to a finite-dimensional central simple k-algebra A, and Quillen [Q] showed that the K-theory of V is given as

$$K_*(V) = \bigoplus_{n=0}^{\dim V} K_*(A^{\otimes n}).$$

This computation provided an important step in the argument of Merkurjev and Suslin; they also needed to compute some of the K-cohomology groups of V.

Quillen's computations have been generalized to the case of forms of Grassmannians and flag varieties by I. Panin ([P1], [P2]) and independently by ourself, jointly with V. Srinivas and J. Weyman [LSW]. The answer is similar: one gets a sum of the K-groups of tensor powers of a central simple algebra associated to the form, with the sum indexed by certain representations of the general linear group. The K-theory of quadrics has been computed by Swan [Sw] and independently by Kapranov [K]. Here as well, representation theory plays a role, this time that of the spinor group via the Clifford algebra of the quadratic form.

The computation of the K-theory of quadrics has had consequences for understanding the Milnor K_3 of a field. Rost [R] and, independently, Merkurjev and Suslin [MS2] have used Swan's results to prove a Hilbert's theorem 90 for Milnor K_3, with respect to quadratic extensions. Using this, they relate the mod 2 Milnor K_3 to mod 2 Galois cohomology. The quadrics arise as compactifications of the varieties solving the equation

$$\mathrm{Nrd}(x) = c,$$

where Nrd is the reduced norm map of a quaternion algebra over k, and c is an element of k^*.

A question related to the computation of the K-groups of homogeneous spaces is the computation of the K-groups of the algebraic groups themselves. A particu-

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