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GENERIC RIGOROUS ASYMPTOTIC EXPANSIONS FOR WEAKLY NONLINEAR MULTIDIMENSIONAL OSCILLATORY WAVES

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1. Introduction. This note is devoted to the proof that, generically, formal oscillatory waves described by asymptotic expansions of infinite order using the WKB method, define rigorous approximate solutions close to exact oscillatory solutions of semilinear or quasi-linear hyperbolic systems.

Since we deal with the general case in which nonlinear interactions of oscillatory waves take place, we have to make some strong assumptions on the set of phases modelling the oscillations. A basic and natural one is the *coherence assumption* of [HMR] which provides a sound base for the WKB method, assessing a formal control of the first error term.

Indeed, without assumptions on the set of phases, it is known that everything goes badly. In particular, the Cauchy problem for initial oscillatory data is not, in general, well posed in several space dimensions, as is shown in [JMR3, 4]. This phenomenon is typical of space dimension greater than 1. It is due to direct or nonlinear indirect *focusing* effects leading to a blowup of L^∞ norms.

The *coherence* hypothesis or the *oscillating plane* hypothesis (more in the spirit of the one space dimension analysis, [JMR5]) are strong geometrical assumptions preventing phases from focusing. As a consequence [JMR3, 4, 5], the oscillatory Cauchy problem is well posed, and exact solutions are described asymptotically by the leading term of a formal asymptotic expansion.

This note is aimed at describing rigorously situations when a *complete expansion* is available. Toward that goal the simplest policy is followed. Unlike [JMR3, 4], where coherence is the only hypothesis, we choose to add, in the most natural way, technical assumptions as soon as they may help.

When formally deriving equations for the profiles (Section 2), differentiation with respect to phases displays a linear differential operator $P(x, \partial_\theta)$ in the fast variables. Its symbol is the restriction of the symbol of the system on the lagrangian manifold defined by the phases. As in [JMR2] for one space dimension, the coherence assumption (Section 3) implies solvability of $PU = F$, U and F being periodic profiles, at least in the category of formal trigonometric series.

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