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A TRACE FORMULA FOR SYMMETRIC SPACES

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1. Introduction.

1.1. Let G be reductive group defined over an algebraically closed field F and let H be the fixator of an involution θ of G. Roughly speaking, the space of double cosets $H\backslash G/H$ is parametrized by the conjugacy classes in another group G' (see [R], [KR]).

In more detail, assume G semisimple and simply connected. Let $\tau: G \to G$ be the map defined by $\tau(g) = g\theta(g)^{-1}$ and S the image of τ. Clearly, τ is constant on the left cosets of G modulo H and induces an isomorphism σ from G/H to S. Then $\sigma(xgH) = x\sigma(gH)x^{-1}$ for $x \in H$. Thus the double cosets of $H\backslash G/H$ correspond via σ to the adjoint orbits of H on S. The closed H-orbits are those of the semisimple elements contained in S. They can be described as follows. Let A be a maximal θ-stable torus contained in S. All such tori are conjugate under the action of H. Then every closed orbit of H in S intersects A. Furthermore, there is a Chevalley restriction theorem for the situation at hand. Indeed, let $F[S]$ and $F[A]$ be the rings of regular functions on S and A respectively. Let H^0 be the neutral connected component of H and let W_H be the quotient of the normalizer of A in H^0 by the centralizer of A in H^0. Then the restriction map

$$F[S] \to F[A]$$

restricts to an isomorphism

$$F[S]^H \simeq F[A]^{W_H}.$$

Since $F[S]^H$ separates the closed orbits, we can identify the set of closed orbits of H in S to the (maximal ideal) spectrum of the algebra $F[A]^{W_H}$, that is, to the orbits

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