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LONG-RANGE ONE-PARTICLE SCATTERING IN A HOMOGENEOUS MAGNETIC FIELD

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1. Introduction. There has been extensive research on the long-range one-particle scattering using both the stationary methods and phase-space analysis (see e.g., [1]–[3], [5], [6], [9], [11]–[13], [15]–[17], [25], [26], [28], and references given there). By comparison, considerably less is known about scattering in the presence of a magnetic field. In the case of one particle in a constant (homogeneous) field, which is the subject of this paper, asymptotic completeness for short-range potentials was shown in [4] by stationary (trace-class and Agmon-Kuroda) methods. The short-range decay was actually required only in the direction of the field, in other directions any rate of decay and, in the azimuthally symmetric case, even considerable (e.g., polynomial) growth was allowed. This result was recovered by Simon ([32]) who used the original phase-space analysis of Enss ([10]). In [4] the existence and completeness of the modified wave operators were proved for Coulomb potentials; however, the azimuthal symmetry of the latter played a crucial role in the proof which therefore could not be extended to the general long-range case. In this paper we will make assumptions only on decay (and not on symmetry) of the potential.

To fix notation, let $r = (x, y, z)$ be the coordinates in $\mathbb{R}^3$ and let the magnetic field be constant and parallel to the z axis. The trajectory of a classical charged particle, provided no other forces are present, will be a spiral—the superposition of uniform motion parallel to the direction of the field and circular motion in the (x, y) plane. Its quantum equivalent is the superposition of free motion in the z direction and a bound state called a Landau orbit (or a linear combination of such) in the (x, y) plane. Suppose that a potential decaying at infinity as $|r|^{-\mu}$, where $\mu > 0$ and $r^2 = x^2 + y^2 + z^2$, is switched on. If $\mu > 1$, one expects that its influence is negligible as the particle escapes to infinity, so that the trajectory can be approximated well by motion in the magnetic field only as described above. This was indeed proved in [4]. In the long-range case ($0 < \mu \leqslant 1$), the influence of the potential at large distances can no longer be ignored. Thus the motion of the particle in the z direction will approach not a free trajectory, but one given by the solution to the corresponding equations of classical mechanics (see (2.13)–(2.16)). However, the motion in the transversal plane is still expected to converge to a combination of bound states. The propagation along the z axis can be controlled by the usual tools of

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