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TOPOLOGICAL INVARIANTS OF AFFINE HYPERSURFACES: CONNECTIVITY, ENDS, AND SIGNATURE

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1. Introduction. Let V_0 be a hypersurface in $\mathbf{C}^{n+1}$ given by a polynomial equation $P(z_1, \ldots, z_{n+1}) = 0$. Suppose that V_0 has an isolated singularity at a point. In this situation the topology of the part $V_{t,\varepsilon}$ of a perturbed hypersurface $P = t$ (for small t) inside of a ball B_ε of radius ε about the singular point is well understood. In particular, $V_{t,\varepsilon}$ is an $(n-1)$-connected parallelizable $2n$-manifold which comes with a monodromy action on the middle-dimensional homology arising when t varies around a small circle about zero in a t-plane. Along these lines, one obtains a beautiful construction of exotic spheres [Br] and rational homology spheres which appear as the boundaries $\partial V_{t,\varepsilon}$ of $V_{t,\varepsilon}$. The invariants determining the type of $\partial V_{t,\varepsilon}$, such as the above mentioned monodromy or the signature of the intersection form on the middle-dimensional homology of $V_{t,\varepsilon}$, can be obtained in some cases directly from the equation P. In particular for the signature, in the case when P is weighted homogeneous, one obtains a combinatorial formula [Br] related to the Dedekind sums [HZ].

The purpose of this paper is to study another situation associated with a polynomial of several variables which exhibits a similar behavior. To be more precise, under certain conditions the affine hypersurface $P(z_1, \ldots, z_{n+1}) = t$ for $|t| > N$ is an $(n-1)$-connected $2n$-manifold for which the essential part of its topology is encoded in the intersection form and in the monodromy action on the n-dimensional homology which is induced by going around a circle of a large radius in the t-plane (monodromy at infinity). In particular, one obtains a construction of homology and homotopy spheres as the ends of affine polynomial hypersurfaces.

The topology of affine hypersurfaces was studied recently by Broughton [Brt] who discovered a class of polynomials, which he called the tame polynomials and for which the affine hypersurface $P = t$ is an $(n-1)$-connected $2n$-manifold. Previously, an estimate for the connectivity of affine hypersurfaces was obtained by M. Kato [Ka]. Our class of polynomials is defined in terms of a resolution of the base points of the pencil of hypersurfaces which are closures in $\mathbf{P}^n$ of the hypersurfaces $P(z_1, \ldots, z_{n+1}) = t$. Moreover, we describe a wider class of polynomials for which the homology groups of corresponding hypersurfaces vanish in all dimensions except the middle one (Section 2). The examples of polynomials for which the end of $P = t$ is a homology and homotopy sphere are given in Section 3. The analysis

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