

THE VARIATIONAL CONTRIBUTION OF THE PERIODIC
ORBITS PROVIDED BY THE BIRKHOFF-LEWIS
THEOREM (WITH APPLICATIONS TO
CONVEX HAMILTONIANS AND TO
THREE-BODY-TYPE PROBLEMS)

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To your soul, my dear and beloved father.

May your blessing extend over my family, and may you forgive me.

“Do not despair of the appeasing breath of God. For those who
despair of this breath of God are only the Nation of Nonbelievers.”

—The Koran

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0. Introduction. Hamiltonian systems can be studied by different approaches. They are dynamical systems which preserve a symplectic form. Therefore, especially in precise examples, sharp analytic studies of such systems can be completed. Among these kinds of studies, we can find Alekseev’s work on the restricted three-body problem ([25]) or the Birkhoff-Lewis theorem ([1], [2]). Two powerful tools, of general type, describe this approach very well: symbolic dynamics and the Poincaré-Birkhoff theorem (see [26]).

On the other hand, the trajectories of Hamiltonian systems are extremals of the action functional; and, therefore, a variational approach for the study of these

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