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PERIODIC SOLUTIONS TO SOME N-BODY TYPE PROBLEMS: THE FIXED ENERGY CASE

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1. Introduction and statement of the results. In this paper we seek periodic solutions of a conservative system

$$-m_i\ddot{u}_i = \sum_{j=1, i\neq j}^{n} \nabla V_{ij}(u_i - u_j) \tag{1}$$

satisfying, for a fixed real h, the energy equation

$$\frac{1}{2}\sum_{i=1}^{n} m_i|\dot{u}_i|^2 + \frac{1}{2}\sum_{i,j=1, i\neq j}^{n} V_{ij}(u_i - u_j) = h. \tag{2}$$

Concerning the potentials V_{ij}, we assume that $(\forall i \neq j)$

$$V_{ij}(x, t) = V_{ji}(-x, t) \qquad \forall x \in \mathbf{R}^k\backslash\{0\}, \tag{V1}$$

$$V_{ij}(x, t) \leqslant 0 \qquad \forall x \in \mathbf{R}^k\backslash\{0\}, \tag{V2}$$

$$\lim_{x\to 0} -|x|^2 V_{ij}(x) = +\infty, \tag{V3}$$

$$\begin{gathered}\exists \rho > \rho_0 > 0 \exists \theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}: \\ \operatorname{ang}(\nabla V_{ij}(x, t), x) \leqslant \theta \qquad \forall x, 0 < |x| < \rho_0 \quad \text{and} \quad \forall |x| > \rho,\end{gathered} \tag{V4}$$

$$\lim_{x\to 0} |x|\,|\nabla V_{ij}(x)| = +\infty. \tag{V5}$$

Our goal is the following result.

THEOREM 1. *Assume* (V1)–(V5) *holds. Then for every* $h > 0$, *problem* (1)–(2) *has at least one periodic solution* $u = (u_1, \ldots, u_n)$ *such that* $u_i(t) \neq u_j(t)$, $\forall t \in \mathbf{R}$, $\forall i \neq j$.

Assumption (V1) corresponds to Newton's third law of mechanics. It is a necessary condition for the variational formulation of the problem when written in the

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