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LOCAL EXISTENCE AND STABILITY OF MULTIVALUED SOLUTIONS TO DETERMINED ANALYTIC FIRST-ORDER SYSTEMS ON THE PLANE

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Introduction. The objective of this paper is to establish the local existence and stability of specific types of multivalued solutions to determined, nonlinear, analytic, first-order PDEs on the plane (see (22)). Our main result is that at regular points existence is controlled by the classical characteristic variety and, at type-changing points, existence is controlled by transversality conditions of the type encountered in Thom-Boardman singularity theory. (This is in contrast to [K] where the existence of multivalued immersed Lagrangian solutions was controlled by a microlocal invariant of the PDEs *independent* of elliptic, parabolic, or hyperbolic type. Essentially, this is because the multivalued solutions considered here are of higher order; i.e., here there exist well-defined limiting 2-jets (versus 1-jets) at singular points, or more geometrically, here Lagrangian solutions have nonimmersed points. This contrast is discussed in Examples (a)(ii) and (c).) As an application, multivalued solutions will be used to establish the local existence of specific types of weak single-valued solutions. In geometric applications these multivalued solutions are of interest in their own right since their graphs correspond to smooth 2-surfaces in 4-space with well-behaved nonimmersed points (i.e., map germs $\mathbf{R}^2 \to \mathbf{R}^4$ with immersive analytic lifts to the Grassmann bundle of 2-planes over $\mathbf{R}^4$). Such map germs are inherently unstable. However, we will see that relative to a fixed regular PDE the multivalued solutions constructed in this paper are stable in the space of multivalued solution germs.

In Section I we review the geometry of the Grassmann bundle of 2-planes in 4-space $\pi\colon G_2(\mathbf{R}^4) \to \mathbf{R}^4$ and its canonical exterior differential system $\mathscr{I}$. We then define the ambient characteristic variety on $G_2(\mathbf{R}^4)$. In Lemma 1 we show that for rank-0 transverse $\mathscr{I}$-integral surfaces $s\colon D \subset \mathbf{R}^2 \to G_2(\mathbf{R}^4)$, $s^*\mathscr{I} = 0$, the interaction of s and the ambient characteristic variety determines the basic differential topology of its graph $\pi \circ s\colon D \to \mathbf{R}^4$. (In this regard, Lemma 1(c) resembles a multivalued analogue of the Eisenbud-Levine degree formula [EL].) In Observation 1 we excise from such an s a moderately regular single-valued function $F\colon \mathbf{R}^2_d \to \mathbf{R}^2_r$ whose graph lies in $\pi \circ s(D) \subset \mathbf{R}^4 \cong \mathbf{R}^2_d \times \mathbf{R}^2_r$. In Lemma 2 we show that for rank-1 transverse $\mathscr{I}$-integral surfaces an integer-valued contact invariant determines the basic differential topology of the graph. In Observation 2 we excise a moderately regular single-valued function from such an integral surface.

In Section II we view a PDE as a codimension-2 variety $\Sigma^6 \subset G_2(\mathbf{R}^4)$ and show

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