Vol. 69, No. 3 DUKE MATHEMATICAL JOURNAL © March 1993

# CHARACTERS, DUAL PAIRS, AND UNITARY REPRESENTATIONS

TOMASZ PRZEBINDA

**0. Introduction.** The main inspiration for this work is an open problem of constructing irreducible unitary representations of classical groups, attached to nilpotent coadjoint orbits. At present, it is not clear what the word "attached" means. We would like to suggest an approach motivated by Howe's description of the oscillator representation $\omega$ in terms of the Weyl transform and the Cayley transform [H2] (see 1.7). This description implies immediately a "Cayley-Kirillov-Rossmann"-type character formula for each of the irreducible pieces $\omega_+$, $\omega_-$ of $\omega$, where the Fourier transform of character is supported on the closure of a single nilpotent coadjoint orbit [P1, (5.4), (6.7)]. Thus the representations $\omega_+$ and $\omega_-$ are attached to this orbit in a classical, easily acceptable way. This phenomenon persists for a number of other irreducible unitary representations of classical groups [P1] (see 6.13), but we do not follow this (thorny) path in this work. Instead, we concentrate on the associated varieties and the wave front sets.

Let $W$ be a symplectic space over $\mathbf{R}$ and let $G$, $G'$ be an irreducible dual pair in the symplectic group $Sp = Sp(W)$, [H7]. Let $\mathfrak{g}$, $\mathfrak{g}'$ denote the Lie algebras of $G$, $G'$ respectively. There are canonical moment maps (see (2.6))

$$\tau_{\mathfrak{g}}: W \to \mathfrak{g}^*, \qquad \tau_{\mathfrak{g}'}: W \to \mathfrak{g}'^*, \tag{0.1}$$

which intertwine the actions of $G$, $G'$ on $W$ with the coadjoint actions on $\mathfrak{g}^*$, $\mathfrak{g}'^*$ respectively. Here is an interesting and easily verifiable [H9] property of these maps:

$$\begin{aligned} &\tau_{\mathfrak{g}'}(\tau_{\mathfrak{g}}^{-1}(\text{a nilpotent coadjoint orbit in } \mathfrak{g}^*)) \\ &\quad = \text{union of nilpotent coadjoint orbits in } \mathfrak{g}'^*. \end{aligned} \tag{0.2}$$

The maps (0.1) extend canonically to the complexifications

$$\tau_{\mathfrak{g}}: W_{\mathbf{C}} \to \mathfrak{g}_{\mathbf{C}}^*, \qquad \tau_{\mathfrak{g}'}: W_{\mathbf{C}} \to \mathfrak{g}_{\mathbf{C}}'^* \tag{0.3}$$

and intertwine the appropriate actions of the complexified algebraic groups $G_{\mathbf{C}}$, $G'_{\mathbf{C}}$. The first fundamental theorem of the classical invariant theory asserts that $\tau_{\mathfrak{g}}$, $\tau_{\mathfrak{g}'}$ are quotient maps (under $G_{\mathbf{C}}$, $G'_{\mathbf{C}}$), in the sense of algebraic geometry [KP1, 2.2],

Received 29 May 1992. Revision received 23 July 1992.
Author's research partially supported by NSF Grant DMS 9000938.