Vol. 69, No. 2 DUKE MATHEMATICAL JOURNAL © February 1993

GLOBAL CRYSTAL BASES OF QUANTUM GROUPS

MASAKI KASHIWARA

0. Introduction.

0.1. In [K$_2$], we constructed the global crystal bases of $U_q^-(\mathfrak{g})$ and of the irreducible $U_q(\mathfrak{g})$-modules with highest weight. The purpose of this article is to construct the global crystal basis of the q-analogue $A_q(\mathfrak{g})$ of the coordinate ring of the reductive algebraic group associated with the Lie algebra $\mathfrak{g}$. The idea of construction is similar to [K$_2$]. By the q-analogue of the Peter-Weyl theorem, $A_q(\mathfrak{g})$ has a decomposition $\bigoplus_\lambda V(\lambda)^* \otimes V(\lambda)$ as a bi-$U_q(\mathfrak{g})$-module, where $V(\lambda)$ is the irreducible $U_q(\mathfrak{g})$-module with a dominant integral weight λ as highest weight. Hence $A_q(\mathfrak{g})$ has a (upper) crystal base $(L(A_q(\mathfrak{g})), B(A_q(\mathfrak{g}))) = \bigoplus(L(\lambda)^*, B(\lambda)^*) \otimes (L(\lambda), B(\lambda))$ at $q = 0$ and similarly a crystal base $(\bar{L}(A_q(\mathfrak{g})), \bar{B}(A_q(\mathfrak{g})))$ at $q = \infty$ (see §7 for their normalization). We denote by $U_q^{\mathbf{Q}}(\mathfrak{g})$ the sub-$\mathbf{Q}[q, q^{-1}]$-algebra of $U_q(\mathfrak{g})$ generated by $e_i^{(n)}$, $f_i^{(n)}$, q^h, and $\begin{Bmatrix} q^h \\ n \end{Bmatrix}$. We denote by $\langle\ ,\ \rangle\colon A_q(\mathfrak{g}) \times U_q(\mathfrak{g}) \to \mathbf{Q}(q)$ the canonical pairing, and we define

$$A_q^{\mathbf{Q}}(\mathfrak{g}) = \{u \in A_q(\mathfrak{g}); \langle u, U_q^{\mathbf{Q}}(\mathfrak{g})\rangle \subset \mathbf{Q}[q, q^{-1}]\}.$$

Then $A_q^{\mathbf{Q}}(\mathfrak{g})$ is a subalgebra of $A_q(\mathfrak{g})$ satisfying $A_q(\mathfrak{g}) = \mathbf{Q}(q) \otimes_{\mathbf{Q}[q,q^{-1}]} A_q^{\mathbf{Q}}(\mathfrak{g})$.

Now the main result of this article is the following.

THEOREM 1. (i) *Set* $E = A_q^{\mathbf{Q}}(\mathfrak{g}) \cap L(A_q(\mathfrak{g})) \cap \bar{L}(A_q(\mathfrak{g}))$. *Then* $E \to L(A_q(\mathfrak{g}))/qL(A_q(\mathfrak{g}))$ *is an isomorphism, and* $A_q^{\mathbf{Q}}(\mathfrak{g}) = \mathbf{Q}[q, q^{-1}] \otimes_{\mathbf{Q}} E$.

(ii) *Letting G be the inverse of the isomorphism above, we have*

$$A_q^{\mathbf{Q}}(\mathfrak{g}) = \bigoplus_{b \in B(A_q(\mathfrak{g}))} \mathbf{Q}[q, q^{-1}]G(b).$$

0.2. Theorem 1 is a consequence of the following theorem, Theorem 2.

Let M be an integrable $U_q(\mathfrak{g})$-module with highest weights and $M_{\mathbf{Q}}$ a sub-$U_q^{\mathbf{Q}}(\mathfrak{g})$-module of M such that $Q(q) \otimes_{\mathbf{Q}[q,q^{-1}]} M_{\mathbf{Q}} \cong M$. Let (L_0, B_0) and (L_∞, B_∞) be an upper crystal base of M at $q = 0$ and $q = \infty$, respectively. Let $H = \{u \in M; e_i u = 0$ for any $i\}$ be the set of highest-weight vectors.

THEOREM 2. *Assume the following conditions*:

(i) $\{u \in M; e_i^{(n)} u \in M_{\mathbf{Q}}$ *for any* i *and* $n \geqslant 1\} = M_{\mathbf{Q}} + H$;

(ii) $H \cap M_{\mathbf{Q}} \cap L_0 \cap L_\infty \to (H \cap L_0)/(H \cap qL_0)$ *is an isomorphism.*

Received 24 March 1992. Revision received 31 August 1992.