

VARIATIONS OF THE MIXED HODGE STRUCTURE OF AFFINE HYPERSURFACES IN ALGEBRAIC TORI

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1. Introduction. The study of infinitesimal variations of Hodge structure on cohomology of algebraic varieties developed by J. Carlson, M. Green, P. Griffiths, and J. Harris has been originated from the fundamental paper of Griffiths on periods of complex hypersurfaces in projective spaces [31]. The paper of Griffiths contains a very fruitful approach to a description of the Hodge structure on the primitive part PH^{n-1} of the middle cohomology group of projective hypersurfaces $\bar{Z}_f \subset \mathbf{P}^n$ defined by homogeneous polynomials $f(Y) \in \mathbf{C}[Y_0, Y_1, \dots, Y_n]$. Griffiths has proved the existence of an isomorphism between the quotients of the Hodge filtration on $PH^{n-1}(\bar{Z}_f)$ and the homogeneous components of an Artinian graded ring R_f . The ring R_f is called a *Jacobian ring*. It is a quotient of the polynomial ring $\mathbf{C}[Y_0, Y_1, \dots, Y_n]$ by the *Jacobian ideal* J_f generated by a regular sequence consisting of partial derivatives $\partial f(Y)/\partial Y_i$, ($0 \leq i \leq n$).

One should remark that a similar idea was also discovered earlier by Dwork in his p -adic proof of the Weil's conjectures for hypersurfaces in projective spaces [20, 21]. Instead of projective hypersurfaces, Dwork has considered only *affine parts* Z_f

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