Vol. 69, No. 2   DUKE MATHEMATICAL JOURNAL ©   February 1993

ON THE STANDARD L-FUNCTION FOR G_2

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Let $G = G_2$. The group ${}^LG^0 = G_2(\mathbb{C})$ has a seven-dimensional irreducible representation which we refer to as the standard representation of G_2. Let π be an irreducible generic cusp form on $G(\mathbb{A})$. According to the Langlands program we may associate to π and to the standard representation of G_2 an L-function which we call the standard L-function (see Section 3).

In [15] a Rankin-Selberg integral representation is constructed for this L-function. This integral involves an Eisenstein series on SO_7 induced from a non-maximal parabolic.

In this paper we construct another Rankin-Selberg integral which represents the standard L-function of G_2. This construction involves an Eisenstein series on the double cover of SL_2. Since the poles of this Eisenstein series are well known (see [1]), this enables us to show that the partial standard L-function can have at most one simple pole. Finally, we show that the existence of this pole implies a non-vanishing property of a certain period (see Section 5).

I wish to thank S. Rallis for useful conversations, and D. Soudry for explaining to me parts of the local theory.

1. Notations

(1.1) Let $G = G_2$. G has two simple roots, α the short root and β the long root. Its positive roots are denoted by $\alpha, \beta, \alpha + \beta, 2\alpha + \beta, 3\alpha + \beta, 3\alpha + 2\beta$. The long roots β, $3\alpha + \beta$, $3\alpha + 2\beta$ form the root system of SL_3. If ε is a root, $x_\varepsilon(r)$ will denote the one parametric subgroup corresponding to ε.

Let $P = GL_2U$ (resp. $Q = GL_2V$) be the maximal parabolic subgroup of G such that $x_\alpha(r) \subset GL_2$ (resp. $x_\beta(r) \subset GL_2$). Thus $\dim U = \dim V = 5$. We shall denote by R the maximal unipotent radical of G. The maximal split torus of G is denoted by $h(t_1, t_2)$ and parametrized such that

$$h^{-1}(t_1, t_2)x_\alpha(r)h(t_1, t_2) = x_\alpha(t_2^{-1}r)$$

$$h^{-1}(t_1, t_2)x_\beta(r)h(t_1, t_2) = x_\beta(t_1^{-1}t_2r).$$

Under this embedding of SL_3 in G, $h(t_1, t_2)$ is identified with $\mathrm{diag}(t_1, t_2, t_1^{-1}t_2^{-1})$. The two simple reflections of the Weyl group of G, corresponding to α and β, are denoted by w_α and w_β.

Received 27 April 1992. Revision received 11 August 1992.
Author partially supported by NSF 9103263.