

## THE $GL(3)$ RANKIN-SELBERG CONVOLUTION FOR FUNCTIONS NOT OF RAPID DECAY

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**0. Introduction.** There has been a great deal of recent research involving the Fourier coefficients of metaplectic Eisenstein series (e.g. [BH], [BFH], [BFH2], [GHP], [L], [L2]). Eisenstein series have been discovered with Fourier coefficients which are (essentially) the Hecke L-series of the cubic residue symbol, the L-series of quadratic twists of elliptic curves with complex multiplication, the L-series of cubic twists of elliptic curves with complex multiplication, and the L-series of quadratic twists of cuspidal newforms for the group  $\Gamma_0(M)$ . Once a form with interesting Fourier coefficients is found, it is often desirable to obtain analytic information about the average behavior of the Fourier coefficients by studying L-series associated to the form. Such investigations can lead to theorems yielding average values for, and obtaining the nonvanishing of, the Fourier coefficients.

In the case of metaplectic forms, such L-series in general do not have Euler products, and to obtain the desired analytic information about them it is necessary to apply variations of the Rankin-Selberg method. This method has been studied in great generality by Jacquet, Piatetskii-Shapiro, and Shalika (generalizing the earlier work of Jacquet-Langlands) and is well understood in a large number of cases when the forms involved are cuspidal, i.e. of rapid decay, but other than these and a few cases analogous to a slow decay Mellin transform, there is only one example in the literature of a convolution involving forms not of rapid decay. This is the case of  $GL(2)$ ; it was worked out (independently) by Zagier and Patterson [Z, P]. Unfortunately, in all the interesting examples known on the metaplectic group, the forms involved are Eisenstein series and thus are not of rapid decay.

We study here the first treatment of a  $GL(3)$  Rankin-Selberg convolution where the forms being convolved are not cuspidal forms. We will state a result for the convolution of two  $GL(3)$  forms that are not of rapid decay and also for an

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