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ON THE DECOMPOSITION OF A REPRESENTATION OF $GL(3)$ RESTRICTED TO $GL(2)$ OVER A P-ADIC FIELD

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It is a theorem of J. Bernstein that, given irreducible admissible representations V of $GL(n)$ and W of $GL(n-1)$, the space of $GL(n-1)$-equivariant maps from V to W has dimension at most one where $GL(n-1)$ sits in $GL(n)$ as the subgroup $\begin{pmatrix} A & 0 \\ 0 & 1 \end{pmatrix}$. Here and in the rest of the paper, $GL(n)$ will denote the group of k-rational points of the corresponding algebraic group over a fixed nonarchimedean local field k. In this paper we give a complete classification of those irreducible admissible representations V of $GL(3)$, and W of $GL(2)$, such that there is a nonzero $GL(2)$-equivariant map of V onto W, and make a conjecture for the general n.

This paper is being written essentially as an exercise in dealing with the extension problems which arise in the Mackey theory about restriction of an induced representation to a subgroup. However, as the reader will see, we have been lucky here in the sense that certain extensions which might have created a problem have been taken care of by some other considerations.

I would like to thank Y. Flicker for posing the question of characterisation of $GL(3)$-representations with $GL(2)$-invariant forms, i.e., the case corresponding to $W = \mathbf{C}$ (and more generally for $GL(n)$). He himself obtained a complete classification of those *unitary* representations of $GL(n)$ which have $GL(n-1)$-invariant form; see [F1].

1. Notation and other preliminaries. For a representation ρ of $GL(n)$ and a character χ of $GL(1)$, we let $\rho \cdot \chi$ denote the representation of $GL(n)$ obtained by twisting ρ by χ: $\rho \cdot \chi(x) = \chi(\det x)\rho(x)$. In particular, a character of k^* will also be thought of as a character of $GL(n)$. We let ν denote the character $\nu(x) = |\det x|$ of $GL(n)$ or of any of its subgroups.

For any locally compact group G, δ_G will denote the positive square root of its modulus function.

We will always use normalised induction in this paper (and so $\mathrm{ind}_H^G \pi$ will have the "extra factor" δ_G/δ_H). Let us note that $\mathrm{Ind}_P^{GL(3)}[\rho \otimes \chi]$, where $P = MN$ denotes the standard maximal parabolic of $GL(3)$ of type (2, 1), and $\rho \otimes \chi$ is a representation of $M = GL(2) \times GL(1)$, is the space of functions f on $GL(3)$ with values in the representation space of ρ such that $f(p \cdot g) = \delta_P^{-1}(m)\chi(m)\rho(m)f(g)$ where $p = mn$, and $\delta_P(m) = |\det A|^{-1/2}|d|$ for $m = \begin{pmatrix} A & 0 \\ 0 & d \end{pmatrix}$, where A is a 2×2 matrix.

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