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FREE PRODUCTS OF HYPERFINITE VON NEUMANN ALGEBRAS AND FREE DIMENSION

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Introduction. Voiculescu's theory of freeness in noncommutative probability spaces (see [10, 11, 12, 13, 14, 15], especially the latter for an overview) has made possible the recent surge of results about and related to the free-group factors $L(\mathbf{F}_n)$ [13, 3, 7, 8, 9, 4]. One hopes to eventually be able to solve the old isomorphism question of whether $L(\mathbf{F}_n) \cong L(\mathbf{F}_m)$ for $n \neq m$.

In Voiculescu's theory, (see also [1]), one takes free products of finite von Neumann algebras, denoted $\mathcal{A} * \mathcal{B}$, and one has $L(G) * L(H) \cong L(G * H)$, (where $L(G)$ for G a discrete group is the group von Neumann algebra, which is generated by the left regular representation of G on $l^2(G)$). It is of intrinsic interest to decide when $\mathcal{A} * \mathcal{B}$ is a factor and to determine its isomorphism class. It may be that such results or the techniques used will give insight into the isomorphism problem. Moreover, such free products are related to amalgamated free products, which have arisen in connection with results about irreducible subfactors [6, 9, 1′].

In [4] and [9], the interpolated free-group factors $L(\mathbf{F}_r)$ $(1 < r \leqslant \infty)$ were found, that have equality with the free-group factor on n generators if $r = n \in \mathbf{N}\backslash\{0, 1\}$ and that satisfy

$$L(\mathbf{F}_r) * L(\mathbf{F}_{r'}) = L(\mathbf{F}_{r+r'}) \qquad (1 < r, r' \leqslant \infty) \tag{1}$$

and

$$L(\mathbf{F}_r)_\gamma = L\left(\mathbf{F}\left(1 + \frac{r-1}{\gamma^2}\right)\right) \qquad (1 < r \leqslant \infty, 0 < \gamma < \infty). \tag{2}$$

Moreover, it was seen that these two formulas imply that either all the $L(\mathbf{F}_r)$ are isomorphic to each other or that no two of them are isomorphic.

In this paper, we examine free products of general hyperfinite von Neumann algebras and express the answer in terms of the interpolated free-group factors. (A finite von Neumann algebra is one which has an everywhere-defined faithful trace-state. It is hyperfinite if it has a dense subalgebra which is the increasing union of finite-dimensional algebras; i.e., for us "hyperfinite" means finite approximately finite-dimensional. Since by [5] there is only one hyperfinite II_1-factor R, every hyperfinite von Neumann algebra is a direct sum of R tensor an abelian algebra

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