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THE DIFFERENTIAL FORM SPECTRUM OF MANIFOLDS OF POSITIVE CURVATURE

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Introduction. In this paper we consider spectral properties of the Hodge Laplacian $\Delta_p = d\delta + \delta d$ acting on L^2 differential forms of a complete noncompact Riemannian manifold M with nonnegative sectional curvatures. The topological structure of these manifolds is described by the theorem of Cheeger and Gromoll [CG]: M contains a compact submanifold S without boundary which is totally convex (in particular, connected and totally geodesic) and whose normal bundle NS is diffeomorphic to M. In general, S is not unique, and the diffeomorphism is not given by the normal exponential map of S. This last condition can be verified in many examples and will be our main technical assumption in this paper. If S is zero-dimensional, this reduces to the assumption that S is a *pole* of M—i.e., that the exponential map from S is a diffeomorphism onto M.

Our main results are based on some Rellich-type identities for differential forms, similar to the main identity proved in [EF] for eigenfunctions of the Laplacian. Under quadratic curvature decay conditions similar to those in [EF], these identities allow us to prove the vanishing of L^2 harmonic forms in certain degrees. In the rotationally symmetric nonnegatively curved case, vanishing in all degrees was proved in [Dod]; for 1-forms it holds in great generality [GW1]. (We recall the precise statements in Section 2.) Denote by $\mathscr{H}^p(M)$ the space of p-forms $u \in L^2\Omega^p(M)$ such that $du = \delta u = 0$ (equivalently, such that $\Delta_p u = 0$). Our vanishing theorem follows.

THEOREM 2.1. *Let M be a complete noncompact n-dimensional Riemannian manifold of nonnegative sectional curvatures ($n \geqslant 3$). Assume M has a soul S of dimension $s \leqslant n-2$ such that* $\exp_S$*: $NS \to M$ is a diffeomorphism. (If $s = 0$, we take this to mean S is a pole of M.) Let $0 < p < (n-s)/2$ or $(n+s)/2 < p < n$ and assume the radial sectional curvatures satisfy*

$$0 \leqslant K_r \leqslant \frac{c(1-c)}{r^2}$$

on $M - S$, where r denotes distance to S and $(2p-1)/(n-s-1) < c < 1$. Then $\mathscr{H}^p = \{0\}$.

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