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A STABILITY CRITERION FOR BIORTHOGONAL WAVELET BASES AND THEIR RELATED SUBBAND CODING SCHEME

ALBERT COHEN AND INGRID DAUBECHIES

Introduction. In a recent work with J. C. Feauveau [CDF], we introduced biorthogonal bases of compactly supported wavelets, i.e., pairs of dual Riesz bases generated from two single compactly supported functions ψ and $\tilde{\psi}$ by means of dilations and translations

$$\begin{cases} \psi_k^j(x) = 2^{-j/2}\psi(2^{-j}x - k) & (j, k) \in \mathbb{Z}^2 \\ \tilde{\psi}_k^j(x) = 2^{-j/2}\tilde{\psi}(2^{-j}x - k) & (j, k) \in \mathbb{Z}^2. \end{cases} \tag{1.1}$$

This construction mimics, in a more general setting, the construction of orthonormal bases of compactly supported wavelets developed in [Dau1] that we briefly recall here in three steps:

- Orthonormal wavelets are associated with a scaling function φ which defines a multiresolution analysis, i.e., a ladder of embedded approximation subspaces of $L^2(\mathbb{R})$

$$\{0\} \to \cdots V_1 \subset V_0 \subset V_{-1} \cdots \to L^2(\mathbb{R}) \tag{1.2}$$

such that $\{\varphi_k^j\}_{k\in\mathbb{Z}} = \{2^{-j/2}\varphi(2^{-j}x - k)\}_{k\in\mathbb{Z}}$ is an orthonormal basis for V_j. The wavelets are built to characterize the missing details between two adjacent levels of approximation. More precisely, $\{\psi_k^j\}_{k\in\mathbb{Z}} = \{2^{-j/2}\psi(2^{-j}x - k)\}_{k\in\mathbb{Z}}$ is an orthonormal basis for the orthogonal complement W_j of V_j in V_{j-1}.

- The constructions of φ and ψ are based on a trigonometric polynomial $m_0(\omega)$ such that $m_0(0) = 1$ and

$$|m_0(\omega)|^2 + |m_0(\omega + \pi)|^2 = 1. \tag{1.3}$$

The functions φ and ψ are then defined by

$$\hat{\varphi}(\omega) = \prod_{k=1}^{+\infty} m_0(2^{-k}\omega) \tag{1.4}$$

and

$$\hat{\psi}(\omega) = m_1\left(\frac{\omega}{2}\right)\hat{\varphi}\left(\frac{\omega}{2}\right) = e^{-i\omega/2}\,\overline{m_0\left(\frac{\omega}{2} + \pi\right)}\,\hat{\varphi}\left(\frac{\omega}{2}\right). \tag{1.5}$$

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