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ON THE EXISTENCE OF CLOSED GEODESICS ON NONCOMPACT RIEMANNIAN MANIFOLDS

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1. Introduction. The problem of the existence of a closed geodesic on compact Riemannian manifolds $\mathcal{M}$ has been faced by several authors (see e.g. [7, 9]).

The only result that we know about the noncompact case is a theorem of Thorbergsson (see [18]) where the existence of a closed geodesic is proved when $\mathcal{M}$ is not contractible and its sectional curvature is nonnegative outside some compact set. In that paper the author proves the existence of a compact convex subset of $\mathcal{M}$ and studies the existence problem in such a compact subset of $\mathcal{M}$. This is possible if the sectional curvature at infinity is nonnegative.

In this paper, we prove the existence of at least one closed geodesic on the noncompact Riemannian manifold $\mathcal{M}$—under different assumptions on its topology and its sectional curvature—directly working on the entire manifold $\mathcal{M}$. In particular, we suppose that the *lim sup* of the curvature at infinity is nonpositive.

Let $\mathcal{M}$ be a Riemannian manifold. We shall denote by $\Lambda(\mathcal{M})$ the free loop space on $\mathcal{M}$ (with the compact-open topology) and by $K(x)$ ($x \in \mathcal{M}$) the supremum of the sectional curvatures, i.e.,

$$K(x) = sup\{K_\pi : \pi \subset T_x\mathcal{M}\} \tag{1.1}$$

where $T_x\mathcal{M}$ is the tangent space of $\mathcal{M}$ at x and K_π is the sectional curvature with respect to the plane $\pi \subset T_x\mathcal{M}$.

Moreover, we denote by $d(\cdot, \cdot)$ the distance induced by the Riemannian structure of $\mathcal{M}$.

We can now state the main result of the paper.

THEOREM 1.1. *Let $(\mathcal{M}, \langle\ ,\ \rangle)$ be a complete, connected, noncompact C^∞-Riemannian manifold. Assume that*

(i) *there exists an integer $q > 2 \cdot dim\, \mathcal{M}$ such that*

$$H_q(\Lambda(\mathcal{M}), \mathbb{K}) \neq 0,$$

where $H_q(\cdot, \mathbb{K})$ is the qth singular homology group with coefficients in a field $\mathbb{K}$;

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