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HAUSDORFF PERTURBATIONS OF RICCI-FLAT MANIFOLDS AND THE SPLITTING THEOREM

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0. Introduction. The purpose of this paper is twofold. First, further examples are constructed of compact manifolds of positive, bounded, and ε-pinched Ricci curvature, in the spirit of previous constructions in [2], [24], [25]. As in [2], these constructions are based on gravitational instantons, but instead of the Gibbons-Hawking metrics used there, we use here the Riemannian Schwarzschild metric and various higher-dimensional generalizations. In this way, the construction and examples of Sha-Yang [25] are recovered, more or less exactly. In particular, the construction of Sha-Yang [25] can be reformulated in terms of appropriate glueings of gravitational instantons (or their generalizations), consistent with the approach in [2]. This approach gives the construction a more conceptual underpinning, and also has the advantage of producing certain infinite families of manifolds with ε-pinched Ricci curvature.

Our main motivation however is to exhibit counterexamples to several questions on the topological rigidity associated to the Cheeger-Gromoll splitting theorem [12]. We also discuss some implications for the study of degenerations of Riemannian manifolds with a uniform bound on Ricci curvature.

To put this in perspective, recall the following theorem.

THEOREM 0.1 (Toponogov splitting) [26]. *If M is a complete manifold of nonnegative sectional curvature, $K_M \geqslant 0$, and $L \approx \mathbb{R}$ is a line in M, i.e., a complete length minimizing geodesic in M, then M splits isometrically as a product*

$$M = \mathbb{R} \times M'. \tag{0.1}$$

This global rigidity result has a corresponding local or finite version, which implies the local rigidity of Riemannian manifolds with a lower bound on the curvature, in neighborhoods of minimizing geodesic segments.

More precisely, let $\gamma\colon [-L, L] \to M$ be a length-minimizing normal geodesic in a Riemannian manifold M. Given a free (small) parameter $a > 0$, define

$$N_a(\gamma) = \{x \in M\colon x = \exp_{x_o} v, \text{ for } \langle \gamma', v\rangle = 0, \|v\| \leqslant a, \text{ and } x_o \in \gamma[-L\cdot a, L\cdot a]\}. \tag{0.2}$$

$N_a(\gamma)$ will be called the normal neighborhood of radius a about $\gamma|_{[-L\cdot a, L\cdot a]}$.

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