Vol. 67, No. 3 DUKE MATHEMATICAL JOURNAL © September 1992

CLIFFORD ALGEBRAS AND POLAR PLANES

GUDLAUGUR THORBERGSSON

1. Introduction. We will associate a polar plane to a representation of the Clifford algebra C_{m-1} on $\mathbb{R}^l$ or equivalently to an $(m+1)$-tuple $(P_0, \ldots, P_m)$ of symmetric $(2l \times 2l)$-matrices satisfying

$$P_iP_j + P_jP_i = 2\delta_{ij}\,\mathrm{Id}.$$

Such an $(m+1)$-tuple is called a symmetric Clifford system in [FKM].

These polar planes have a natural topology that is compatible with their incidence structure. If $m \neq 1$, 2, and 4, they are not Moufang. If m is 1, 2, or 4, we get known examples.

Another way to state our result is to say that the simplicial complex $\Delta(M)$ associated to an isoparametric hypersurface M of Clifford type is a Tits building of type C_2. See [PT2] and [Th2] for the definitions of these concepts.

Let Σ be the unit sphere in the linear space of the symmetric matrices spanned by $P_0, \ldots, P_m$, and set

$$\mathscr{P} := \{x \in S^{2l-1} \mid \langle Px, x\rangle = 0 \text{ for all } P \in \Sigma\}$$

and

$$\mathscr{L} := \{\ell(x, P) \mid x \in \mathscr{P} \text{ and } P \in \Sigma\}$$

where

$$\ell(x, P) := \{y \in \mathscr{P} \mid P(x-y) = -(x-y)\}.$$

We will call the elements of $\mathscr{P}$ *points* and those of $\mathscr{L}$ *lines*. If $x \in \ell$ for $x \in \mathscr{P}$ and $\ell \in \mathscr{L}$, then we say that "p is incident to ℓ", "p lies on ℓ", "ℓ contains p", etc. We then prove the following theorem.

THEOREM. *The pair $(\mathscr{P}, \mathscr{L})$ satisfies the axioms of a polar plane:*

(1) *Any two points are incident to at most one line.*

(2) *Let p be a point and ℓ a line not containing p. Then there is a unique line ℓ' that meets ℓ and contains p.*

Received 10 May 1991. Revision received 11 February 1992.
Partially supported by NSF Grant DMS-8901443