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A WARPED PRODUCT SPLITTING THEOREM

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The celebrated Cheeger-Gromoll splitting theorem (see [ChGr] and also [EsHe, Wu]) implies that a connected, complete Riemannian manifold having nonnegative Ricci curvature and two ends is isometric to a Riemannian product, where one factor is a line. There is a version of this theorem for compact, connected Riemannian manifolds M with two smooth boundary components. It states that, if M has nonnegative Ricci curvature and two boundary components, both of which have nonnegative mean curvature with respect to their inward normals, then M is isometric to the Riemannian product of N_1 with a closed interval (in particular, N_1 is isometric to N_2).

In this paper we will prove an analog of this result for manifolds with Ricci curvature $\geqslant -(n-1)$. Since our argument parallels the proof of the splitting theorem for manifolds with nonnegative Ricci curvature, we present the proofs simultaneously by letting δ be either zero or one in what follows.

THEOREM 1. *Let M be a connected, complete, n-dimensional Riemannian manifold with two boundary components N_1 and N_2, at least one of which is compact. Suppose Ricci$(M) \geqslant -(n-1)\delta$, the mean curvature of N_1 is $\geqslant \delta$, and the mean curvature of N_2 is $\geqslant -\delta$. Then M is isometric to $N_1 \times [0, l]$ with the warped product metric $ds^2 = e^{2\delta t} g_0 + dt^2$, where g_0 is the metric on N_1. Moreover, g_0 has nonnegative intrinsic Ricci curvature.*

Putting the second boundary component N_2 at infinity, we get the following theorem.

THEOREM 2 ("Cusp", or "halfspace" rigidity). *Let M be a complete, n-dimensional Riemannian manifold with compact, smooth boundary N. Suppose*

1. *Ricci$(M) \geqslant -(n-1)\delta$;*
2. *H_N, the mean curvature of N with respect to the inward normal, satisfies $H_N \geqslant \delta$;*
3. *M is noncompact.*

Then M is isometric to $N \times [0, \infty)$ with the warped product metric $ds^2 = e^{2\delta t} g_0 + dt^2$, where g_0 has nonnegative intrinsic Ricci curvature.

Using a slight modification of the proof of Theorem 1, we get the following result for any lower bound on the Ricci curvature.

THEOREM 3. *Let M be a complete, connected Riemannian manifold with two boundary components N_1 and N_2, one of which is compact, and let $\overline{M}$ be an annular*

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