Vol. 67, No. 3 DUKE MATHEMATICAL JOURNAL © September 1992

STABLE RANK-2 BUNDLES ON SIMPLY CONNECTED ELLIPTIC SURFACES

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Introduction. Let X be a simply connected and minimal elliptic surface with the geometric genus p_g and let the multiplicities of the multiple fibers be equal to p and q. For a polarization L on X, let $\mathcal{M}_L(c)$ be the moduli space of all L-stable rank-2 bundles whose first and second chern classes are equal to zero and c respectively. Choose L to be c-suitable (see [6]). In this work, we address the following question.

Question A. What is the structure of $\mathcal{M}_L(c)$ *if* $2 \leqslant c < 2(1 + p_g)$?

When $p_g = 0$, the moduli space $\mathcal{M}_L(1)$ were studied by Donaldson [4], Friedman and Morgan [8, 9], and Okonek and Van de Ven [16]; Lübke and Okonek [14] classified $\mathcal{M}_L(1)$ for any p_g. In general, there exist several difficulties. First of all, the nonemptiness criterions of Taubes [18], Gieseker [11], and Friedman [7] fail when $c < 2(1 + p_g)$. Second, even when $\mathcal{M}_L(c)$ is nonempty, there may exist irreducible components in $\mathcal{M}_L(c)$ which do not have the expected dimension $4c - 3(1 + p_g)$, or have the correct dimension but not reduced.

If $c < 2(1 + p_g)$ and $\mathcal{M}_L(c)$ is nonempty, then all bundles in $\mathcal{M}_L(c)$ fall into case (A) in [6]. Using Friedman's results, we immediately obtain an upper bound for the dimension of $\mathcal{M}_L(c)$. On the other hand, if both p and q are at least three, then we can construct an irreducible component whose dimension is exactly the upper bound. Our first result is the following (see notations below).

THEOREM B. *Assume that* $p, q \geqslant 3$ *and* $2 \leqslant c < 2(1 + p_g)$. *Then the moduli space* $\mathcal{M}_L(c)$ *is nonempty with dimension* $(2c + \lfloor c/2 \rfloor - 1)$.

In particular, we see that every irreducible component in the moduli space $\mathcal{M}_L(1 + 2p_g)$ has the expected dimension, that is, $(1 + 5p_g)$. We classify all irreducible components in $\mathcal{M}_L(1 + 2p_g)$ when $p_g \geqslant 3$ (see Theorem 2.11 for details).

THEOREM C. *Assume that* $p_g \geqslant 3$ *and that L is a suitable polarization. Then*

(i) *the reductions of irreducible components in the moduli space* $\mathcal{M}_L(1 + 2p_g)$ *have Kodaira dimension* $-\infty$;

(ii) *the number of irreducible components in* $\mathcal{M}_L(1 + 2p_g)$ *is equal to*

$$(p_g \cdot p \cdot g - \delta)/2$$

where δ *is defined to be zero if* $(p_g \cdot p)$ *is even, and to be one if* $(p_g \cdot p)$ *is odd.*

Received 26 December 1991.