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COMPLEX TANGENTS OF REAL SURFACES IN COMPLEX SURFACES

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Introduction. In this paper we study the complex tangents of real surfaces in complex surfaces. More precisely, let M be a closed real surface, i.e., a smooth, compact, two-dimensional manifold without boundary. Given an immersion resp. embedding $\pi: M \to \mathcal{M}$ of M into a complex surface $\mathcal{M}$ (a complex manifold of dimension two), we consider the question to what extent can one simplify the structure of the set of complex tangents of π by a regular homotopy (resp. isotopy) of immersions (resp. embeddings).

Recall that a point $p \in M$ is called a *complex tangent* of π if the tangent space $\pi_*(T_pM)$ is a complex linear subspace (a complex line) in $T_{\pi(p)}\mathcal{M}$. The immersion is *totally real* at every point that is not a complex tangent. An immersion without complex tangents is said to be totally real. When M is orientable and we choose an orientation on M, then every complex tangent p of π is either *positive* or *negative*, depending on whether the orientation on $\pi_*(T_pM)$ induced from T_pM by π_* agrees or disagrees with the canonical orientation of $\pi_*(T_pM)$ as a complex line.

Recall that a *regular homotopy* is a family of immersions $\pi_t: M \to \mathcal{M}$, $t \in [0, 1]$, such that π_t and all its derivatives depend continuously on the parameter t. Immersions π_0 and π_1 are *regularly homotopic* if there exists a regular homotopy connecting π_0 to π_1. If all immersions in the family π_t are embeddings, we call π_t an *isotopy* of embeddings.

Thom's transversality theorem (see [1] or [2]) implies that a generic immersion $\pi: M \to \mathcal{M}$ only has isolated complex tangents, and its double points are transverse self-intersections (normal crossings) that avoid the complex tangents of π. In this paper we shall only study immersions satisfying these properties, and we will not mention this again.

It is well known that one cannot change the complex tangents arbitrarily by a regular homotopy since their number, counted with suitable algebraic multiplicities, is an invariant $I(\pi)$ of the regular homotopy class of the immersion, called the *index* of π (Chern and Spanier [10], Eliashberg and Harlamov [24], Webster [33], and Forstnerič [16]). Before proceeding, we must recall the definition of $I(\pi)$.

First, we recall from [16] and [33] the *index* $I(p; \pi) \in \mathbf{Z}$ of an isolated complex tangent of π. Let U be a small disc neighborhood of p in M. In suitable local holomorphic coordinates (z, w) on $\mathcal{M}$ near $\pi(p)$, the surface $\pi(U)$ is a graph $w = f(z)$ of a smooth complex function f defined near the origin in $\mathbf{C}$, with $\pi(p)$ corresponding

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