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ON L^2-INDEX THEOREMS FOR COMPLETE MANIFOLDS OF RANK-ONE TYPE

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To the memory of Vojislav Avacumović

1. Introduction. Index problems on noncompact manifolds have attracted much interest recently. In this paper we consider a complete Riemannian manifold M, of dimension m, and a generalized Dirac operator, D, on M. (These operators are described in detail in [LM].) This means that D is an elliptic first-order differential operator acting on the smooth sections of a hermitian vector bundle E over M. Now E has a Clifford module structure and a metric connection, ∇, compatible with Clifford multiplication (denoted by $\cdot$), and locally, with any orthonormal frame (F_i) for TM, we have

$$D\sigma = \sum_{i\geq 1} F_i \cdot \nabla_{F_i}\sigma, \qquad \sigma \in C^\infty(E). \tag{1.1}$$

It is easily seen that D with domain $C_0^\infty(E)$ is symmetric in $L^2(E)$ and that D has a unique closed extension, also to be denoted by D. But if in addition E has a ∇-parallel splitting, $E = E^+ \oplus E^-$, such that $F \cdot E^\pm \subset E^\mp$ for $F \in TM$, then $D^+ := D|C_0^\infty(E^+)$ is no longer symmetric. Thus, we can ask whether the unique closed extension, D^+, is a Fredholm operator, or more generally, whether the spaces $\ker D^\pm \cap L^2(E^\pm)$ are finite-dimensional. If so,

$$L^2\text{-ind } D^+ := \dim\ker D^+ - \dim\ker D^- \tag{1.2}$$

is well defined, and we can try to derive an L^2-index formula. The first example of such a formula seems to occur in [APS], for the geometric operators on manifolds with cylindrical ends. Since then, there has been considerable and ongoing interest in such formulas; see, e.g., [ADS], [BaMo], [GL], [M1], [M2], [M3], [M4], [St1], [St2], among others.

Nevertheless, there is still no systematic approach to such theorems. In [B] we made a modest first step towards an abstract setting the applications of which, however, seemed to be limited to manifolds which are asymptotically warped products. The reason for this limitation was that we required a specific limit for the Dirac operator at infinity. In the present paper we relax this requirement considerably. This is possible by generalizing analogously the notion of *regular singular operator* introduced in [BS2] and [B]. The main result is formulated as Theorem 4.4. Thus, we can treat considerably more general manifolds like multiply warped products, which we treat as main example here in Section 5. But our theory also

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