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TWISTED ENDOSCOPY AND REDUCIBILITY OF INDUCED REPRESENTATIONS FOR p-ADIC GROUPS

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1. Introduction. This is the first in a series of papers in which we study the reducibility of representations induced from discrete series representations of the Levi factors of maximal parabolic subgroups of p-adic groups on the unitary axis. The problem is equivalent to determining certain local Langlands L-functions which are of arithmetic significance by themselves. One of the main aims of this paper is to interpret our results in the direction of the parametrization problem by means of the theory of twisted endoscopy for which our method seems to be very suitable. When the representation is supercuspidal, we also study the reducibility of the induced representations off the unitary axis.

Let F be a p-adic field of characteristic zero. Fix a positive integer $n > 1$ and let $\mathbf{G}$ be either of the groups Sp_{2n}, SO_{2n}, or SO_{2n+1}. In all three cases there is a conjugacy class of maximal parabolic subgroups which has GL_n as its Levi factor. Let $\mathbf{P} = \mathbf{MN}$ be the standard parabolic subgroup of $\mathbf{G}$ in this conjugacy class. Then $\mathbf{M} \cong GL_n$. Let σ be a discrete series representation of $M = GL_n(F)$ and, given $s \in \mathbb{C}$, let $I(s, \sigma)$ be the representation of $G = \mathbf{G}(F)$ induced from $\sigma \otimes |\det(\ \ )|^s$. Let $I(\sigma) = I(0, \sigma)$.

Assume first that σ is supercuspidal. Let $A(s, \sigma, w_0)$ be the standard intertwining operator defined by equation (2.1) of Section 2, where w_0 is a representative for $\tilde{w}_0$, the longest element in the Weyl group of $\mathbf{G}$. (See Section 2.) From the general theory of c-functions and R-groups (see [31, 23, 24, 16]), it is clear that $I(\sigma)$ is reducible if and only if $\tilde{w}_0(\sigma) \cong \sigma$ and $A(s, \sigma, w_0)$ is holomorphic at $s = 0$. By the properties of intertwining operators, there exists a unique polynomial $P_\sigma(q^{-s})$ satisfying $P_\sigma(0) = 1$, such that the operator

$$P_\sigma(q^{-2s})A(s, \sigma, w_0) \tag{1.1}$$

is holomorphic and nonzero. Thus if $\tilde{w}_0(\sigma) \cong \sigma$, then $I(\sigma)$ is irreducible if and only if $P_\sigma(1) = 0$.

On the other hand, let $\varphi\colon W_F \to GL_n(\mathbb{C})$ be the conjectural representation of the Weil group W_F of $\bar{F}/F$, parametrizing σ. (See [4, 33].) Then from the general theory developed in [23] we must conjecturally have

$$L(s, \wedge^2 \rho_n \cdot \varphi) = P_\sigma(q^{-s})^{-1}$$

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