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ON GABRIELOV'S REGULARITY CONDITION FOR ANALYTIC MAPPINGS

WIESŁAW PAWŁUCKI

1. Introduction. Let $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Let X and Y denote analytic manifolds over $\mathbb{K}$. (All manifolds are assumed to be finite-dimensional and countable at infinity.) Let $\mathcal{O}_X$ and $\mathcal{O}_Y$ denote the sheaves of germs of analytic functions on X and Y, respectively. Suppose that $\varphi\colon X \to Y$ is an analytic mapping. Let $\varphi^{-1}\mathcal{O}_Y$ denote the inverse image of $\mathcal{O}_Y$ by φ. Then the stalks of $\varphi^{-1}\mathcal{O}_Y$ are given by $(\varphi^{-1}\mathcal{O}_Y)_x = \mathcal{O}_{Y,\varphi(x)}$.

Let $x \in X$. Then φ induces a homomorphism of local algebras $\varphi_x^*\colon \mathcal{O}_{Y,\varphi(x)} \to \mathcal{O}_{X,x}$ and, thereby, a homomorphism of sheaves of local algebras (on X) $\varphi^*\colon \varphi^{-1}\mathcal{O}_Y \to \mathcal{O}_X$. We also have a homomorphism of the completions $\hat{\varphi}_x^*\colon \hat{\mathcal{O}}_{Y,\varphi(x)} \to \hat{\mathcal{O}}_{X,x}$ induced by φ_x^*.

We can consider the following three kinds of rank of φ at x (the invariants of Gabrielov [10]):

$$r_1(x) = \text{the generic rank of } \varphi \text{ near } x,$$

$$r_2(x) = \text{the Krull dimension of } \hat{\mathcal{O}}_{Y,\varphi(x)}/\operatorname{Ker} \hat{\varphi}_x^*,$$

and

$$r_3(x) = \text{the Krull dimension of } \mathcal{O}_{Y,\varphi(x)}/\operatorname{Ker} \varphi_x^*.$$

It is easy to see that $r_1(x) \leqslant r_2(x) \leqslant r_3(x)$.

Following Bierstone and Milman [2, 3, 4], we say that φ is *regular at x (in the sense of Gabrielov)* if $r_1(x) = r_3(x)$. In other words, φ is regular at x if there exists a neighborhood U of x in X and a locally analytic subset S of Y of dimension $r_1(x)$ such that $\varphi(U) \subset S$. (Clearly, one can always take S to be irreducible at $\varphi(x)$.) It follows that regularity at x is an open condition. We say that φ is regular if it is regular at each point $x \in X$.

The above regularity condition was first considered by Gabrielov [10]. Further papers connected with it were published by Milman [25], Becker and Zame [1], Izumi [19, 20], Bierstone with Milman [2, 3, 4], Tougeron [35], and Spivakovsky [34].

Bierstone and Milman [2, 3, 4] apply regular mappings to differential analysis. One of their results is the following solution to the composition problem of Glaeser [12].

Suppose that X and Y are real analytic manifolds and that $\varphi\colon X \to Y$ is a semiproper analytic regular mapping. Let $C^\infty(X)$ denote the Fréchet algebra of C^∞ functions on

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