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GALOIS REPRESENTATIONS ATTACHED TO mod p COHOMOLOGY OF $GL(n, \mathbb{Z})$

AVNER ASH

Let p be a prime number and $\mathbb{F}$ a finite field of characteristic p. We consider the cohomology of $GL(n, \mathbb{Z})$ and its subgroups of finite index with coefficients in finite-dimensional $\mathbb{F}$-vector spaces. Our main interest concerns their structure as modules for the action of the Hecke algebra. When $n = 2$, it follows from Theorem 3.1 below and the theory of Eichler and Shimura (reduced mod p) that we can attach to a Hecke eigenclass a representation of the absolute Galois group $G_{\mathbb{Q}}$ of $\mathbb{Q}$ into $GL(2, \mathbb{F})$, such that the Hecke polynomial at a prime ℓ equals the characteristic polynomial of a Frobenius element above ℓ for almost all primes ℓ. Conversely, by a conjecture of Serre [Se], all odd irreducible representations of $G_{\mathbb{Q}}$ into $GL(2, \mathbb{F})$ are supposed to arise this way.

The purpose of this paper is to propose and investigate the following conjecture generalizing the first of these phenomena when $n > 2$. (Precise definitions of the terms in the conjecture may be found in Section 1.)

CONJECTURE B. *Let $n \geqslant 2$, Γ be a subgroup of finite index in $GL(n, \mathbb{Z})$, and (Γ, S) be a congruence Hecke pair of level N. With p and $\mathbb{F}$ as above, let V be an admissible $\mathbb{F}S$-module. Suppose $\beta \in H^i(\Gamma, V)$ is an eigenclass for the action of the Hecke algebra $H(N)$ with eigenvalues $a(\ell, k) \in \mathbb{F}$.*

Then there exists a continuous semisimple representation $\rho: G_{\mathbb{Q}} \to GL(n, \mathbb{F})$ unramified outside pN such that

$$\sum (-1)^k \ell^{k(k-1)/2} a(\ell, k) X^k = \det(I - \rho(\mathrm{Frob}_\ell)^{-1} X)$$

for all ℓ not dividing pN.

The content of the new conjecture supersedes what one obtains from standard conjectures in the theory of automorphic forms (reduced mod p), essentially by expecting that p-torsion in the integral cohomology will also yield Galois representations.

It would be nice if brand new Galois representations arose from torsion classes in this way, but I do not know if that happens. On the other hand, I view Conjecture B as a means to understand the wide variety of torsion classes that do, in fact, exist.

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