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SCHATTEN-VON NEUMANN CLASSES OF MULTILINEAR FORMS

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0. Introduction. At the Leipzig meeting in September, 1983, Pietsch [12] outlined the foundations of a theory of normed ideals of multilinear forms in Banach spaces. At about the same time one of the present authors had begun to work on the survey [10], where an entire chapter (Chapter V) is devoted to extending the theory of trace ideals of operators in a Hilbert space to the multilinear situation. Both authors proposed the study of a number of questions on Schatten-von Neumann classes S_p of multilinear forms over Hilbert spaces. Their motivations were different. For Pietsch the main reason for developing such a theory was the study of nonlinear operators, while Peetre's motivation was the investigation of Hankel operators.

Later, in [6] and [11] these problems are also mentioned, but the theory of normed ideals of multilinear forms over Hilbert spaces is still in a rather poor state.

This contrasts with the extensively developed theory of trace ideals of operators in a Hilbert space. (See, e.g., the monographs by Gohberg and Krein [5], Simon [14], and Pietsch [13].)

To give the reader an idea of the kind of obstructions one can find when working with multilinear forms, let us mention that such a central tool in the operator case as the Schmidt representation theorem for compact operators is no longer valid in the multilinear context.

In this paper we establish several results about normed ideals of forms. In particular, we settle some problems leftover in the literature cited above.

We start by showing that the dual space of compact forms is the space of nuclear forms and that the dual of the space of nuclear forms is the space of bounded forms. These duality results are given in Section 2.

In Section 3 we characterize Hilbert-Schmidt forms by interpolation, complex and real. Namely, we prove that

$$[S_1, S_\infty]_{1/2} = [S_1, \mathscr{B}]_{1/2} = S_2 \qquad \text{(equal norms)}$$

and that

$$(S_1, S_\infty)_{1/2,2} = (S_1, \mathscr{B})_{1/2,2} = S_2 \qquad \text{(equivalent norms)}$$

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