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FUNDAMENTAL G-STRATA FOR CLASSICAL GROUPS

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Introduction. Let G be (the group of rational points of) a reductive group defined over a local nonarchimedean field k. In recent years considerable effort has been expended in analyzing an irreducible admissible representation π of G, via its restriction to suitable open compact subgroups. This method, which was initiated by R. Howe [H], has thus far been attempted when $G = \mathbb{GL}_N$ or when G has small rank (≤ 2); for examples we refer the reader to [AK], [B], [C], [H], [HM], [K], [KM], [Mo1], [Mo2], [Mo3].

Suppose for the moment that $G = \mathbb{GL}_N$. Then the open compact subgroups that one employs are congruence subgroups arising from parahoric subgroups; in fact, these congruence subgroups arise from filtrations defined by standard affine height functions associated to the affine root system of G. These standard filtrations were first defined and employed (for semisimple G) by Prasad and Raghunathan [PR]. They can also be interpreted (see below) via the filtrations by powers of the Jacobson radicals of the associated hereditary orders when $G = \mathbb{GL}_N$; thus, one has a noncommutative generalization of the standard filtrations employed in algebraic number theory.

Returning to the general philosophy, suppose that G is reductive and that P is a parahoric subgroup of G. Suppose further that $\{P_n\}_{n \geqslant 0}$ is a (yet to be determined) filtration of P by open normal subgroups such that P_n/P_{n+1} is abelian ($n \geqslant 1$), $P_0 = P$, and P_0/P_1 is the group of rational points of a reductive group defined over the residue field of k. If π is an irreducible admissible representation of G, one then looks at $\pi|P$, and the least $n = n(\pi, P)$ such that $\pi|P_{n+1}$ contains a nonzero fixed vector. Then P_n acts on this space of fixed vectors, and one hopes that by varying P and $\{P_n\}$ one can find a "best possible" such $n(\pi, P)$ in some sense and, then, that only a restricted subset of $(P_n/P_{n+1})^\wedge$ can occur which will partially describe π and play the role of "lowest K-types." For this program to succeed, one needs a convenient description of $(P_n/P_{n+1})^\wedge$.

For example, suppose again that $G = \mathbb{GL}_N$. Then P is the group of units of a hereditary order $\mathscr{A}$, and $P_n = 1 + \mathscr{B}^n$, where $\mathscr{B}$ is the Jacobson radical of $\mathscr{A}$ ($n \geqslant 1$); moreover, when $n \geqslant 1$, $(P_n/P_{n+1})^\wedge$ can be identified with $\mathscr{B}^{\lambda(n+1)}/\mathscr{B}^{\lambda(n)}$, where $\lambda(n) = c - n$, c some fixed integer. Thus, each $\psi \in (P_n/P_{n+1})^\wedge$ is associated with a coset $\delta_\psi \in \mathscr{B}^{\lambda(n+1)}/\mathscr{B}^{\lambda(n)}$. Moreover, to P, $\{P_n\}$ is associated an integer $N \geqslant e = e(\{P_n\}) \geqslant 1$. The result associated to the philosophy can now be simply described.

Given π, vary P, $\{P_n\}$, and choose $n(\pi, P)/e(\{P_n\}) = n/e$ as small as possible.

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