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LOCAL ZETA FUNCTIONS AND EULER CHARACTERISTICS

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To the memory of Prof. L. Bouckaert

1. Introduction. Let $K \subset \mathbb{C}$ be a number field, $\mathfrak{O}_K$ the ring of algebraic integers of K, and $\mathfrak{p}$ any maximal ideal of $\mathfrak{O}_K$. We denote the completion of K, resp. $\mathfrak{O}_K$, with respect to $\mathfrak{p}$ by $K_\mathfrak{p}$, resp. $R_\mathfrak{p}$. Reduction mod $\mathfrak{p}$ will always be denoted by $^-$; thus in particular $\bar{K}_\mathfrak{p}$ is the residue field of $K_\mathfrak{p}$. Let q be the cardinality of $\bar{K}_\mathfrak{p}$; thus $\bar{K}_\mathfrak{p} = F_q$. For $x \in K_\mathfrak{p}$ let ord $x \in \mathbb{Z} \cup \{+\infty\}$ be the $\mathfrak{p}$ valuation of x, $|x| = q^{-\operatorname{ord} x}$, and $\operatorname{ac}(x) = x\pi^{-\operatorname{ord} x}$, where $\pi \in \mathfrak{O}_K$ is a fixed uniformizing parameter for $R_\mathfrak{p}$.

Let ψ be a character of $R_\mathfrak{p}^\times$, i.e., a homomorphism $\psi: R_\mathfrak{p}^\times \to \mathbb{C}^\times$ with finite image, where $R_\mathfrak{p}^\times$ denotes the group of units of $R_\mathfrak{p}$. We formally put $\psi(0) = 0$. Let $f(x) \in K[x]$, $x = (x_1, \ldots, x_m)$, $f \neq 0$. To these data one associates Igusa's local zeta function

$$Z(K_\mathfrak{p}, \psi, s) = \int_{R_\mathfrak{p}^m} \psi(\operatorname{ac} f(x))|f(x)|^s |dx|$$

where $|dx|$ denotes the Haar measure so normalized that $R_\mathfrak{p}^m$ has measure one. Let $X = \operatorname{Spec} K[x]$ and $D = \operatorname{Spec} K[x]/(f(x))$. Choose a resolution (Y, h) for f over K, meaning that Y is an integral smooth closed subscheme of projective space over X, $h: Y \to X$ is the natural map, the restriction $h: Y\backslash h^{-1}(D) \to X\backslash D$ is an isomorphism, and $(h^{-1}(D))_{\text{red}}$ has only normal crossings as subscheme of Y. Let E_i, $i \in T$, be the irreducible components of $(h^{-1}(D))_{\text{red}}$. For each $i \in T$ let N_i be the multiplicity of E_i in the divisor of $f \circ h$ on Y and let $\nu_i - 1$ be the multiplicity of E_i in the divisor of $h^*(dx_1 \wedge \cdots \wedge dx_m)$. For $i \in T$ and $I \subset T$ we consider the schemes

$$\mathring{E}_i := E_i \backslash \bigcup_{j \neq i} E_j, \qquad E_I := \bigcap_{i \in I} E_i, \qquad \mathring{E}_I := E_I \backslash \bigcup_{j \in T\backslash I} E_j.$$

When $I = \phi$, we put $E_\phi = Y$.

For any closed subscheme Z of Y we denote the reduction mod $\mathfrak{p}$ of Z by $\bar{Z}$. As in [D] we say that the resolution (Y, h) for f has good reduction mod $\mathfrak{p}$ if $\bar{Y}$ and all $\bar{E}_i$ are smooth, $\bigcup_{i \in T} \bar{E}_i$ has only normal crossings, and the schemes $\bar{E}_i$ and $\bar{E}_j$ have no common components whenever $i \neq j$. Let S be a finite subset of Spec $\mathfrak{O}_K$ such that for all $\mathfrak{p} \notin S$ we have $f \in R_\mathfrak{p}[x]$, $f \not\equiv 0 \bmod \mathfrak{p}$, and the resolution (Y, h) for f

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