Vol. 63, No. 3 DUKE MATHEMATICAL JOURNAL © August 1991

BOUNDARY BEHAVIOR OF INVARIANT METRICS AND VOLUME FORMS ON STRONGLY PSEUDOCONVEX DOMAINS

DAOWEI MA

0. Introduction. In this paper we will give estimates for a class of invariant metrics and volume forms near the boundary of strongly pseudoconvex domains which are more precise than the known estimates. By a metric we shall mean a differential metric. A topological metric is referred to as a distance function.

The metrics considered in this paper are those metrics F that satisfy the following two conditions:

(i) F reduces to the Poincaré metric on the unit disk Δ

$$F_\Delta(z, \xi) = |\xi|/(1 - |z|^2);$$

(ii) F is decreasing under the holomorphic maps

$$F_\Omega(f(z), df(z)\xi) \leqslant F_D(z, \xi), \qquad f \in \mathrm{Hol}(D, \Omega).$$

Here and in the following, $\mathrm{Hol}(D, \Omega)$ denotes the set of holomorphic maps from D to Ω; $\mathrm{Hol}(D_z, \Omega_w)$ denotes the set of maps in $\mathrm{Hol}(D, \Omega)$ that send $z \in D$ to $w \in \Omega$. The condition (ii) above implies that F is invariant under biholomorphic maps.

The class of metrics considered in this paper includes at least the following three metrics:

(a) the Kobayashi metric ([R]), i.e.,

$$F_D^K(z, \xi) = \inf\{|v|: v \in T_0\Delta, \text{ and there exists an } f \in \mathrm{Hol}(\Delta_0, D_z), df(0)v = \xi\};$$

(b) the Carathéodory metric ([C]), i.e.,

$$F_D^C(z, \xi) = \sup\{|v|: v \in T_0\Delta, \text{ and there exists an } f \in \mathrm{Hol}(D_z, \Delta_0), df(z)\xi = v\};$$

(c) the Sibony metric ([S]), i.e.,

$$F_D^S(z, \xi) = \sup\{\langle \mathscr{L}u(z)\xi, \xi\rangle^{1/2}: u \in \mathscr{S}_{z,D}\}.$$

Received 28 September 1990. Revision received 10 December 1990.