BOUNDARY BEHAVIOR OF INVARIANT METRICS AND VOLUME FORMS ON STRONGLY PSEUDOCONVEX DOMAINS

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0. Introduction. In this paper we will give estimates for a class of invariant metrics and volume forms near the boundary of strongly pseudoconvex domains which are more precise than the known estimates. By a metric we shall mean a differential metric. A topological metric is referred to as a distance function.

The metrics considered in this paper are those metrics $F$ that satisfy the following two conditions:

(i) $F$ reduces to the Poincaré metric on the unit disk $\Delta$

$$F_\Delta(z, \xi) = |\xi|/(1 - |z|^2);$$

(ii) $F$ is decreasing under the holomorphic maps

$$F_\Omega(f(z), df(z)\xi) \leq F_D(z, \xi), \quad f \in \text{Hol}(D, \Omega).$$

Here and in the following, $\text{Hol}(D, \Omega)$ denotes the set of holomorphic maps from $D$ to $\Omega$; $\text{Hol}(D_z, \Omega_w)$ denotes the set of maps in $\text{Hol}(D, \Omega)$ that send $z \in D$ to $w \in \Omega$. The condition (ii) above implies that $F$ is invariant under biholomorphic maps.

The class of metrics considered in this paper includes at least the following three metrics:

(a) the Kobayashi metric ([R]), i.e.,

$$F^K_\Omega(z, \xi) = \inf\{|v|: v \in T_0\Delta, \text{ and there exists an } f \in \text{Hol}(\Delta_0, D_z), df(0)v = \xi\};$$

(b) the Carathéodory metric ([C]), i.e.,

$$F^C_\Omega(z, \xi) = \sup\{|v|: v \in T_0\Delta, \text{ and there exists an } f \in \text{Hol}(D_z, \Delta_0), df(z)\xi = v\};$$

(c) the Sibony metric ([S]), i.e.,

$$F^S_\Omega(z, \xi) = \sup\{\langle L u(z)\xi, \xi\rangle^{1/2} : u \in \mathcal{H}_{z,D}\}.$$