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THE END BEHAVIOR OF COMPLETE 2-DIMENSIONAL AREA-MINIMIZING MOD 2 SURFACES IN $\mathbb{R}^n$

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1. Introduction. Suppose $S \subseteq \mathbb{R}^n$ is a (possibly nonorientable) 2-dimensional surface with unoriented boundary. S is *area-minimizing mod* 2 if S has least area amongst all surfaces, orientable or nonorientable, with the same boundary. If S is complete, then we say S is minimizing mod 2 if each compact piece of S is minimizing mod 2. In geometric measure theory, the "surfaces" considered for this question are the *rectifiable flat chains mod* 2 ([Mo1, §11], [W]). A priori such a surface can be highly singular, but in fact a minimizing 2-dimensional rectifiable flat chain mod 2 S is very regular ([Mo2]): near any interior singularity $p \in S \subseteq \mathbb{R}^n$, S consists of the union of at most $n/2$ smooth, embedded minimal surfaces intersecting orthogonally at p. Consequently, if S is complete, then $S = x(M)$ can be considered as a complete regular minimal immersion of some (possibly disconnected) surface M.

The key to the proof of this regularity theorem is the tangent cone behavior of a mod 2 minimizer ([Mo2, Corollary 6]). The tangent cone to $S \subseteq \mathbb{R}^n$ at any $p \in S \subseteq \mathbb{R}^n$ consists of the union of at most $n/2$ orthogonal 2-planes, each taken with multiplicity 1. The same result holds for the tangent cone at ∞, and in §2 we use this result along with the universal mass bound of [Mo3] to study the end behavior of $S = x(M)$. In Lemma 1 we prove that, at ∞, $x(M)$ can be written as a union of minimal graphs, each graph written off of one of the tangent planes at ∞. This lemma also gives suitable convergence at ∞, from which it follows that each end of M is conformally a punctured disk (when considered in oriented isothermal parameters). It follows that M, or its orientable double cover in case M is nonorientable, is a finitely punctured compact Riemann surface and has finite total curvature (Theorem 2).

Once we know that the ends of M are punctured disks, it follows from the Weierstrass representation for the immersion $x(M)$ ([HO, §3]) that each end of $x(M)$ is simple and either flat or catenoid. (See [R].) In §3 we prove that an individual catenoid end is not area minimizing (Theorem 3). Thus all the ends of $x(M)$ are flat, and consequently, if $x(M)$ has n ends, then $x(M)$ lies fully in a $2n$-dimensional affine subspace of $\mathbb{R}^n$ (Corollary 4). It follows that the connected components of M are immersed into orthogonal subspaces (Corollary 5).

In §4 we classify the possible complete mod 2 minimizers $x(M)$ under the assumption that M is connected and of low genus. These results were stated, and for the most part proved, in [R]. We fill the gap here by supplying the proof of the nonexistence of minimizing Klein bottles.

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