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ON CRYSTAL BASES OF THE Q-ANALOGUE OF UNIVERSAL ENVELOPING ALGEBRAS

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To the memory of Professor Michio Kuga who taught me the joy of doing mathematics

CONTENTS

§0. Introduction. The notion of the q-analogue of universal enveloping algebras is introduced independently by V. G. Drinfeld and M. Jimbo in 1985 in their study of exactly solvable models in the statistical mechanics. This algebra $U_q(\mathfrak{g})$ contains a parameter q, and, when $q = 1$, this coincides with the universal enveloping algebra. In the context of exactly solvable models, the parameter q is that of temperature, and $q = 0$ corresponds to the absolute temperature zero. For that reason, we can expect that the q-analogue has a simple structure at $q = 0$. In [K1] we named crystallization the study at $q = 0$, and we introduced the notion of crystal bases. Roughly speaking, crystal bases are bases of $U_q(\mathfrak{g})$-modules at $q = 0$ that satisfy certain axioms. There, we proved the existence and the uniqueness of crystal bases of finite-dimensional representations of $U_q(\mathfrak{g})$ when $\mathfrak{g}$ is one of the classical Lie algebras A_n, B_n, C_n and D_n. K. Misra and T. Miwa ([M]) proved the existence of a crystal base of the basic representation of $U_q(A_n^{(1)})$ and gave its combinatorial description.

The aim of this article is to give the proof of the existence and uniqueness theorem of crystal bases for an arbitrary symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$. Moreover, we globalize this notion. Namely, with the aid of a crystal base we construct a base named the global crystal base of any highest weight irreducible integrable

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