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p-ADIC WHITTAKER FUNCTIONS ON THE METAPLECTIC GROUP

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In this paper we will denote by $\widetilde{Sp}(2n)$ or $G\widetilde{Sp}(2n)$ the metaplectic double cover of $Sp(2n)$ or $GSp(2n)$ over a local field or an adele ring. We will prove a formula for the nonramified Whittaker functions on $\widetilde{Sp}(2n, F)$, where F is a local field of odd residue characteristic. This formula is similar to the formula for nonramified Whittaker functions on reductive algebraic groups found by Shintani [16] (in the case of $GL(n)$), by Kato [12], and by Casselman and Shalika [9]. The Casselman-Shalika formula has been a cornerstone for many investigations of automorphic L-functions, such as the work of Shahidi on functional equations and the works of a number of authors on the Rankin-Selberg method. It is to be expected that our formula will also have applications.

This paper has its origins in the observation of Maass that the metaplectic $\widetilde{GL}(2)$ Eisenstein series have quadratic Dirichlet L-functions as Fourier coefficients. In [3], [4] we gave a first generalization of this fact, showing that the Whittaker-Fourier coefficients of Eisenstein series on the double cover of $GSp(4)$ contain quadratic twists of degree two L-functions. This phenomenon was exploited in our papers [3], [4], and [5] to obtain nonvanishing theorems for automorphic forms on $PGL(2)$ and their derivatives.

As we shall explain, the present work shows that this phenomenon will generalize further: quadratic twists of degree n L-functions may be found in the Whittaker-Fourier coefficients of Eisenstein series on the double cover of $GSp(2n)$. This work is also the justification for the experiments in [6] and other similar unpublished calculations, where we show that it may be profitable to apply various Rankin-Selberg integrals, especially those representing the spin L-functions on $GSp(2n)$, to Eisenstein series on $G\widetilde{Sp}(2n)$.

The reason that we are able to obtain our explicit formula is that *principal series representations on* $\widetilde{Sp}(2n)$ *have unique Whittaker models.* To put this fact in context, suppose that G is a split reductive group over a local field F. Let χ be a character of the maximal torus T of G. We extend χ to the Borel subgroup B of G and form the induced representation $\mathrm{Ind}(\chi)$. It is well known that if χ is *regular* (i.e., not stabilized by any element of the Weyl group, acting on T and its characters by conjugation) then $\mathrm{Ind}(\chi)$ is irreducible. Moreover, whether or not χ is regular, $\mathrm{Ind}(\chi)$ has a unique Whittaker model. That is, if U is the maximal unipotent subgroup of B and θ is a nondegenerate character of U, we may also form the induced space

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