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SEMISIMPLICITY OF THE GALOIS REPRESENTATIONS ATTACHED TO DRINFELD MODULES OVER FIELDS OF "FINITE CHARACTERISTICS"

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§0. Introduction. In this and a subsequent paper, we prove the semisimplicity of the Galois representations attached to Drinfeld modules.

Let K be a finitely generated extension of a finite field of transcendence degree one. Fix once and for all a place ∞ of K and let A be the ring of elements of K which are regular outside ∞. Let F be a field of *finite type over* A (i.e., a ring homomorphism $\gamma: A \to F$ is given, and F is finitely generated over the image of γ as a field). We say that the *characteristic* of (F, γ) is ∞ or $\mathfrak{p}$ according as γ is injective or $\operatorname{Ker}(\gamma)$ is a nonzero prime ideal $\mathfrak{p}$ of A. In the rest of the paper the terminology "characteristic" is used only in this sense, and never in the usual sense. We write, by abuse of notation, $\operatorname{char}(F) = \infty$ or $\mathfrak{p}$ accordingly. Let $\phi: A \to \operatorname{End}_F(\mathbb{G}_a)$ be a Drinfeld module over F of rank r ([2]). For any nonzero prime ideal $v \neq \operatorname{char}(F)$ of A, the v-adic *Tate module* $T_v(\phi)$ ([1], Chap. 1, (4.11)) is associated with ϕ. This is a free A_v-module of rank r, where A_v is the v-adic completion of A. The absolute Galois group $\pi := \operatorname{Gal}(F^{sep}/F)$ of F acts continuously on $T_v(\phi)$. Let K_v be the fraction field of A_v. Our main result is the following

THEOREM (0.1). *Assume that* $\operatorname{char}(F)$ *is finite and* $v \neq \operatorname{char}(F)$. *Then* $T_v(\phi) \otimes_{A_v} K_v$ *is a semisimple* $K_v[\pi]$*-module.*

It is known that such a statement follows from certain finiteness for isomorphism classes. ([4]. See also the Appendix.) Let $f: \phi \to \phi'$ be a separable isogeny of Drinfeld modules over F. If $\operatorname{Ker}(f)(F^{sep}) \simeq \bigoplus_{i=1}^{n} (A/\mathfrak{a}_i)$ as A-modules, where $\mathfrak{a}_i$ are nonzero ideals of A, we define $\deg(f) := \prod_{i=1}^{n} \mathfrak{a}_i$. Then (0.1) follows from

THEOREM (0.2). *Assume that* $\operatorname{char}(F)$ *is finite. Then the number of F-isomorphism classes of Drinfeld modules ϕ' over F such that there exists a separable isogeny $\phi \to \phi'$ over F of degree prime to* $\operatorname{char}(F)$ *is finite.*

The idea of the proof of (0.2), using the theory of modular heights, comes from Zarhin [5].

The plan of the paper is as follows:

§1 contains some elementary facts on polynomial functions which are needed later.

In §2 the differential heights and the modular heights of Drinfeld modules are defined, and the finiteness theorem (0.2) is reduced to the "bounded height theorem".

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