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SOLUTIONS OF THE (GENERALIZED) KORTEWEG-DE VRIES EQUATION IN THE BERGMAN AND THE SZEGÖ SPACES ON A SECTOR

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1. Introduction. In this paper we consider the Cauchy problem for the (generalized) Korteweg-de Vries equation

$$\begin{aligned} \partial_t u + \partial_x^3 u + a(u)\partial_x u = 0, \qquad & (t, x) \in \mathbb{R} \times \mathbb{R}, \\ u(0, x) = \phi(x), \qquad & x \in \mathbb{R}, \end{aligned} \tag{1.1}$$

where $a(\lambda) = \lambda^p$, $p \in \mathbb{N}$ and ϕ is complex valued.

We let $\Delta(\alpha, \beta) = \{z; \alpha - \beta < \arg z < \alpha + \beta\}$, where $\alpha = 0$ or π and $0 < \beta < \pi/2$. We define the Bergman space $B_{\Delta(\alpha,\beta)}$ and the Szegö space $S_{\Delta(\alpha,\beta)}$ on $\Delta(\alpha, \beta)$ as follows.

$$B_{\Delta(\alpha,\beta)} = \{F : F \text{ is analytic on } \Delta(\alpha, \beta),\ \|F\|_{B_{\Delta(\alpha,\beta)}} < \infty\},$$

$$S_{\Delta(\alpha,\beta)} = \{F : F \text{ is analytic on } \Delta(\alpha, \beta),\ \|F\|_{S_{\Delta(\alpha,\beta)}} < \infty\},$$

where

$$\|F\|_{B_{\Delta(\alpha,\beta)}}^2 = \int_{\alpha-\beta}^{\alpha+\beta} \int_0^\infty |F(re^{i\theta})|^2\, r\, dr\, d\theta,$$

$$\begin{aligned} \|F\|_{S_{\Delta(\alpha,\beta)}}^2 &= \int_0^\infty |F(re^{i(\alpha-\beta)})|^2 + |F(re^{i(\alpha+\beta)})|^2\, dr \\ &\cong \sup_{\alpha-\beta<\theta<\alpha+\beta} \int_0^\infty |F(re^{i\theta})|^2\, dr, \end{aligned}$$

where $\cong$ means the two norms are equivalent to each other. We also define

$$B_{\Delta(\alpha,\beta)}^m = \{F \in B_{\Delta(\alpha,\beta)}\colon \|F\|_{B_{\Delta(\alpha,\beta)}^m}^2 = \sum_{j=0}^m \|\partial_z^j F\|_{B_{\Delta(\alpha,\beta)}}^2 < \infty\}$$

and

$$S_{\Delta(\alpha,\beta)}^m = \{F \in S_{\Delta(\alpha,\beta)}\colon \|F\|_{S_{\Delta(\alpha,\beta)}^m}^2 = \sum_{j=0}^m \|\partial_z^j F\|_{S_{\Delta(\alpha,\beta)}}^2 < \infty\}.$$

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